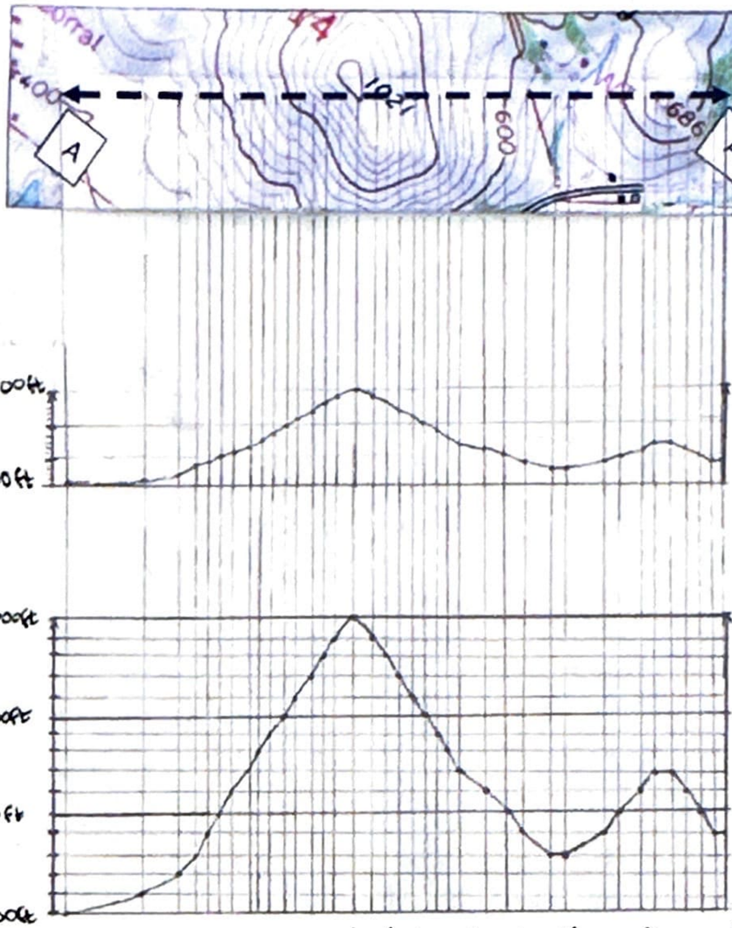
4 / 14 / 21   Geology - 241.02   Mohammad Elassaad

nice touch



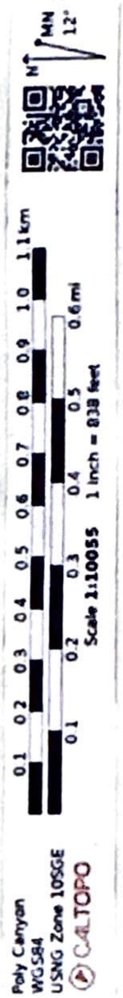
1:1 Ratio

0.5in = 600 ft

3:1

3:1 Ratio

1.5in = 600ft



Label distance across axis, and "square" length otherwise very hard to know if it's 1:1 or 3:1

Note: The bar scale attached to the side of this document was used rather than the singular inch scale due to online / digital distortion.

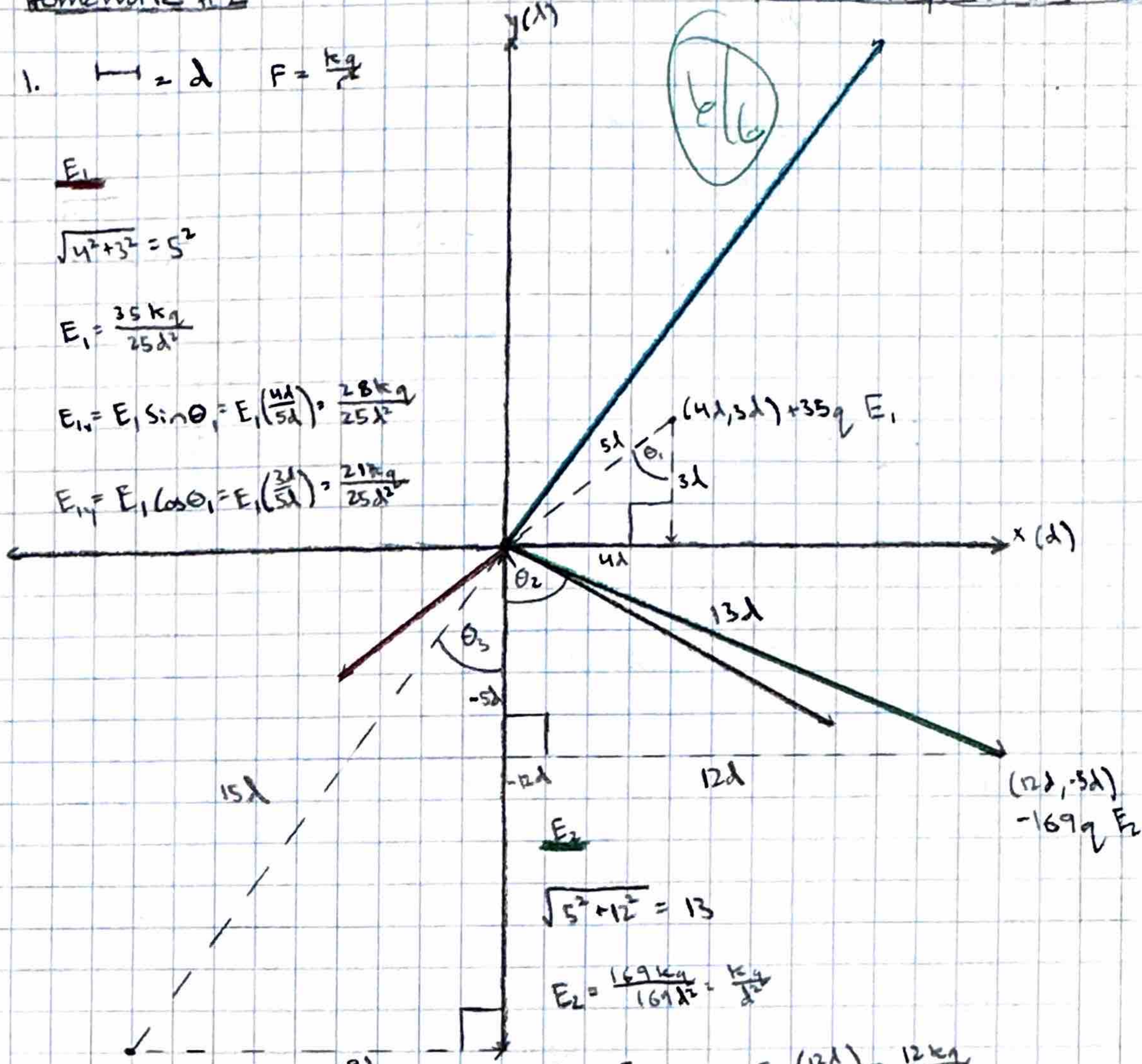
it's a communication matter - you are putting too much burden on reader to see the horizontal axis. This would be esp. difficult in the field

Mohammad Elassaad
Phys 133-54

16/16

Homework #2

1. $\vdash\!\dashv = d \qquad F = \frac{kq}{r^2}$



E_1

$\sqrt{4^2+3^2} = 5^2$

$E_1 = \frac{35kq}{25d^2}$

$E_{1x} = E_1 \sin\theta_1 = E_1\left(\frac{4d}{5d}\right) = \frac{28kq}{25d^2}$

$E_{1y} = E_1 \cos\theta_1 = E_1\left(\frac{3d}{5d}\right) = \frac{21kq}{25d^2}$

E_2

$\sqrt{5^2+12^2} = 13$

$E_2 = \frac{169kq}{169d^2} = \frac{kq}{d^2}$

$E_{2x} = E_2 \sin\theta_2 = E_2\left(\frac{12d}{13d}\right) = \frac{12kq}{13r^2}$

$E_{2y} = E_2 \cos\theta_2 = E_2\left(\frac{5d}{13d}\right) = \frac{5kq}{13r^2}$

E_3

$\sqrt{9^2+12^2} = 15$

$E_3 = \frac{270kq}{225d^2}$

$E_{3x} = E_2 \sin\theta_3 = E_3\left(\frac{9d}{15d}\right) = \frac{18kq}{25d^2}$

$E_{3y} = E_3 \cos\theta_3 = E_3\left(\frac{12d}{15d}\right) = \frac{24kq}{25d^2}$

Net

$x = \frac{kq}{d^2}\left(\frac{18}{25} + \frac{12}{13} - \frac{28}{25}\right) = \frac{34kq}{65d^2}$

$y = \frac{kq}{d^2}\left(\frac{24}{25} - \frac{5}{13} - \frac{21}{25}\right) = -\frac{86kq}{325d^2}$

$\sqrt{x^2+y^2} \approx \frac{29kq}{50d^2}$

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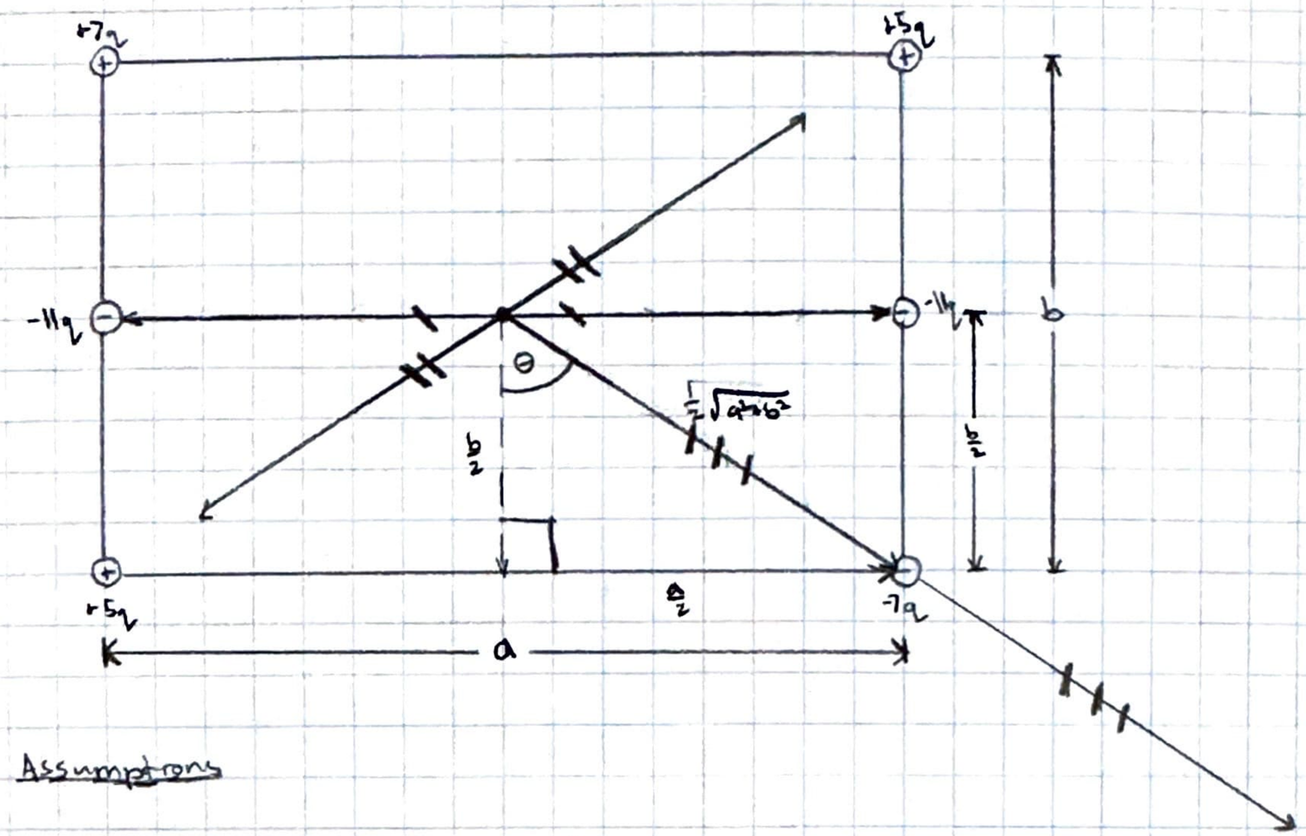
2.



Assumptions

- The -11q charges cancel
- The +5 q charges cancel
- The magnitude of E of the +7q charge creates a vector reaching from the origin to the -7q charge
- Also the ±11q charges should be bigger but I drew them smaller to fit them because they cancel out anyway
- The origin is at the center of a

$$\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2} = \sqrt{\frac{a^2}{4}+\frac{b^2}{4}} = \frac{1}{2}\sqrt{a^2+b^2} = r$$

$$\boxed{\begin{aligned} E &= \frac{kq}{r^2} = 2\left(\frac{7kq}{\left(\frac{1}{2}\sqrt{a^2+b^2}\right)^2}\right) = \frac{14kq}{\frac{1}{4}(a^2+b^2)} = \frac{56kq}{a^2+b^2} \\ E_x &= E\sin\theta = \left(\frac{\frac{a}{2}}{\frac{1}{2}\sqrt{a^2+b^2}}\right)E = \frac{56kqa}{(a^2+b^2)^{\frac{3}{2}}} \\ E_y &= E\cos\theta = \left(\frac{\frac{b}{2}}{\frac{1}{2}\sqrt{a^2+b^2}}\right)E = \frac{56kqb}{(a^2+b^2)^{\frac{3}{2}}} \end{aligned}}$$

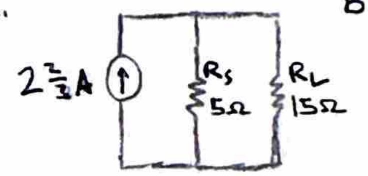
Homework #3

1) $R_{eq} = R_s + R_L = 5 + 15 = 20\Omega$

$V = IR \Rightarrow I = \frac{V}{R} = \frac{10}{20} = \boxed{\frac{1}{2}A}$

$V = (\frac{1}{2})(15) = \boxed{7.5\ \Omega}$

2) a.



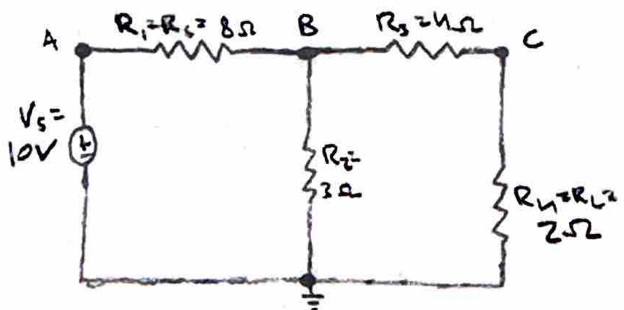
b. $R_{eq} = (\frac{1}{5} + \frac{1}{15})^{-1} = \frac{15}{4}\Omega = \boxed{3.75\Omega}$

$I = \frac{V}{R} = \frac{10}{3.75} = \boxed{2\frac{2}{3}A}$

$V_{RL} = \boxed{10V}$ $\quad I_L = \frac{V}{R} = \frac{10}{15} = \boxed{\frac{2}{3}A}$

c. The voltages and currents for the load are <u>different</u>.

3)



$R_{eq} = 4 + 2 = 6$

$(\frac{1}{6} + \frac{1}{3})^{-1} = 2$

$2 + 8 = 10\Omega$

$I_A = \frac{10}{10} = 1A$

Σi node $B = 0$

$i_{R_1} = i_{R_2} + i_{R_3}$

$1 = \frac{V_2}{R_2} + \frac{V_3}{R_3} \Rightarrow 1 = \frac{2}{3} + \frac{V_3}{4} \Rightarrow V_3 = \frac{4}{3}$

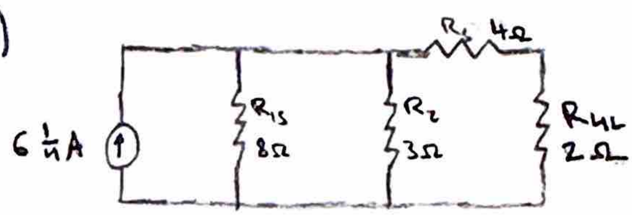
$V = IR = (1)(8) = 8V$

$10 - 8 = 2V = V_2$

$2 - \frac{4}{3} = \boxed{\frac{2}{3}V}$

$I = \frac{2/3}{2} = \boxed{\frac{1}{3}A}$

4)



$R_{eq} = 4 + 2 = 6$

$(\frac{1}{6} + \frac{1}{3})^{-1} = 2$

$(\frac{1}{2} + \frac{1}{8})^{-1} = \boxed{\frac{8}{5}\Omega}$

$I = \frac{V}{R} = \frac{10}{8/5} = \boxed{6\frac{1}{4}A}$

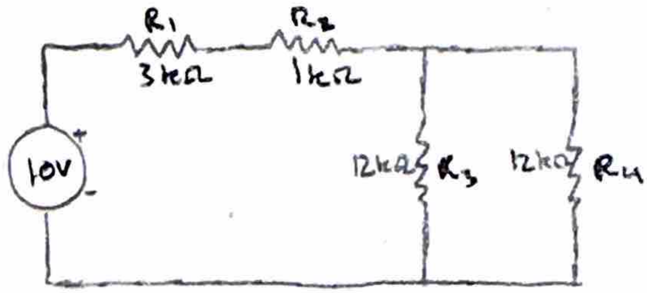
5) $V = IR \Rightarrow I = \frac{V}{R}$

$I_{34} = \frac{10V}{R_{34}} = \frac{10V}{6\Omega} = \boxed{1\frac{2}{3}A}$

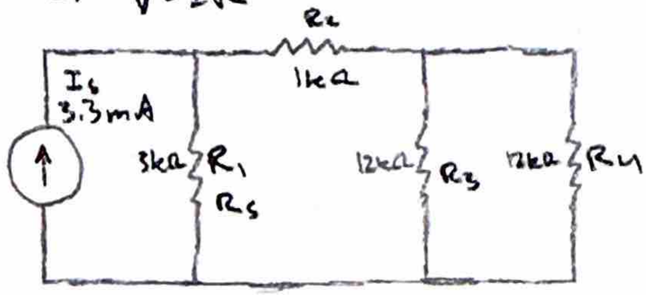
$V = IR = (1\frac{2}{3})(2) = \boxed{3\frac{1}{3}V}$

Homework #10 continued

6.

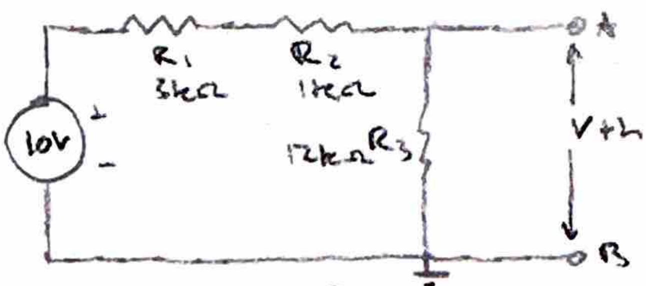


$\Rightarrow V = IR$



$I_s = \frac{V}{R} = \frac{10V}{3000\Omega} = \boxed{3.3mA}$ $\boxed{R_s = 3k\Omega}$

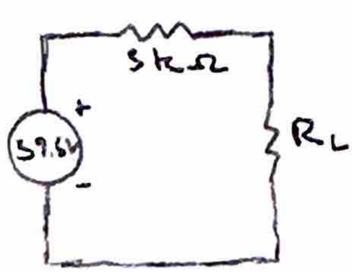
7. $R^{th} = \left(\frac{1}{3+1} + \frac{1}{12}\right)^{-1} = \boxed{3k\Omega = R^{th}}$



$V_{th} = V_A - \cancel{V_B}^{0} = V_A = V_{R_3}$

$I = \frac{V}{R} = \frac{10V}{3000\Omega} = 0.0033A = 3.3mA$

$V_{R_3} = (3.3mA)(12k\Omega) = \boxed{39.6V = V^{th}}$

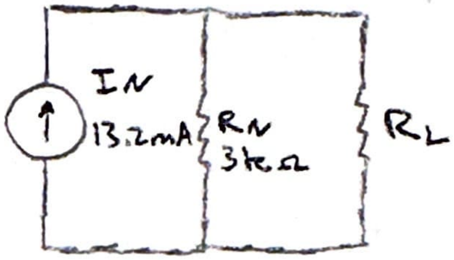


Homework #10 continued

7.

b) $R_N = R_{Th} = \boxed{3\,k\Omega = R_N}$

$I = \frac{V}{R} = \frac{39.6V}{3000\Omega} = 0.0132A = \boxed{I_N = 13.2mA}$



8. $V_c = Ae^{-1/RC^t} \Rightarrow t(99\%) = 5RC$

$t(99\%) = 5 \times 1000\Omega \times 1 \times 10^{-6}F$

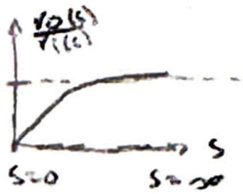
$t(99\%) = 0.005s = \boxed{5ms}$

9. At $s \to 0$, inductor acts as short circuit

$\Rightarrow \frac{V_o(s)}{V_i(s)} = 0$

At $s \to \infty$, inductor acts as open circuit

$\Rightarrow V_o(s) = V_i(s)\left[\frac{R_2}{R_1+R_2}\right]$



$\Rightarrow$ $\boxed{\text{High Pass Filter}}$

10. $F_L = F_r - \frac{1}{2}(BW)$ $\quad F_H = F_r + \frac{1}{2}(BW)$

$BW = \frac{F_r}{Q}$

$F_r = 14\,GHz$ $\quad BW = \frac{2}{2\pi} = 0.3184\,GHz$

a) $F_L = 14 - \frac{1}{2} \times 0.3184 = \boxed{13.8407GHz}$

$F_H = 14 + \frac{1}{2} \times 0.3184 = \boxed{14.15923GHz}$

b) $Q = \frac{F_r}{F_{BW}} = \frac{14}{2/2\pi} = \boxed{43.96 = Q}$

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Midterm 1

1. Max Ice Surface Temp $\Rightarrow$ $T_s = -4°C$

Interface temperature Max = $-9°C = T_{in}$

brine water heat exchanges

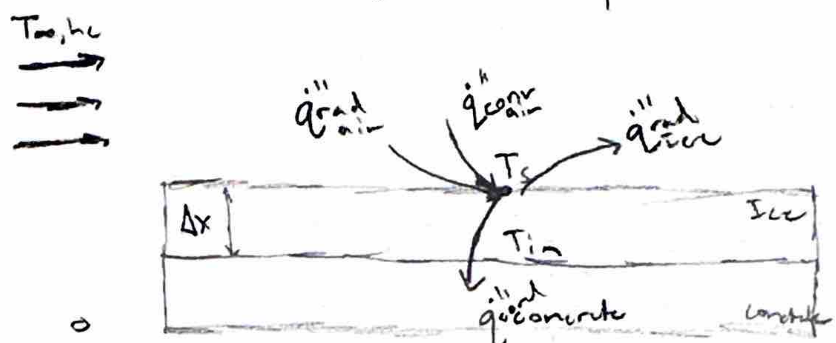
Thickness $\Delta x = 0.1905 m$

Air $T_\infty = 10°C$ $V = 0.5 m/s$ $h_c = 5.0 W/m^2K$

$\varepsilon = 0.8$ $\alpha = 0.55$ $k = 2.30 W/mK$

Total radiant heat flux that is applied to the surface of the ice from the surroundings?

What would be the corresponding surroundings temperature if the surroundings was assumed to be a black body?



$$\cancel{\dot{E}_{ST}}^{\,0} = \cancel{\dot{E}_g}^{\,0} + \dot{E}_{IN} - \dot{E}_{OUT} \Rightarrow \dot{E}_{IN} - \dot{E}_{OUT} = 0$$

Not change with time   No gen

$$q''^{rad}_{air} + q''^{conv}_{air} - \left(q''^{rad}_{ICE} + q''^{cond}_{concrete}\right) = 0$$

$$\Rightarrow \alpha q''^{rad}_{air} + q''^{conv}_{air} - q''^{rad}_{ICE}\varepsilon - q''^{cond}_{concrete} = 0$$

$$q''^{rad}_{Air} = \frac{1}{\alpha}\left(-q''^{conv}_{air} + q''^{rad}_{ICE}\varepsilon + q''^{cond}_{concrete}\right)$$

$h\Delta T$
$5(T_\infty - T_s)$
$5(10-(-4))$
$5(14)$
70

$\varepsilon\sigma T_s^4$
$0.8(5.67\times10^{-8})(269.15)^4$
238.04

$k\frac{\partial T}{\partial x}$
$(2.30)\frac{(-4--9)}{0.1905}$
60.37

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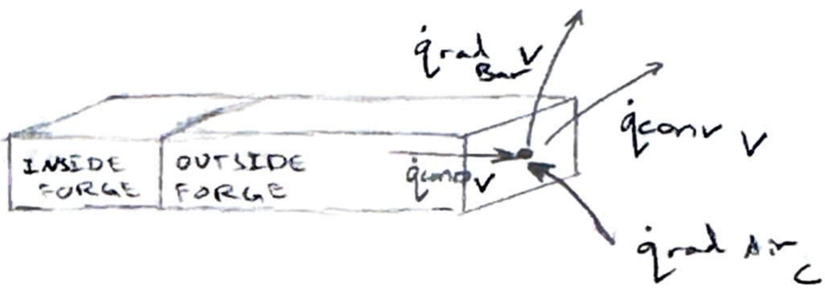

At the tip

$$\dot{E}_{ST} = \dot{E}_{IN} - \dot{E}_{OUT}$$

$J/m^3 s = W/m^3$ $$\rho c \frac{dT}{dt} = \dot{q}'''_{cond} + \dot{q}'''_{rad\,Air} - \dot{q}'''_{rad\,Bar} - \dot{q}'''_{conv}$$

$$\dot{q}'''_{rad\,Air} = \alpha \sigma T_\infty^4 \left(\frac{1}{dx}\right)$$

$$\dot{q}'''_{conv} = h\left(\frac{1}{dx}\right)(T_1 - T_\infty)$$

5.

$$\cancel{\dot{E}_{ST}}^{\,0} = \cancel{\dot{E}_g}^{\,0} + \dot{E}_{IN} - \dot{E}_{OUT}$$

Heat rate $\dot{q}$

$$0 = \dot{E}_{IN} - \dot{E}_{OUT}$$

$$0 = \dot{q}_{cond} + \dot{q}_{rad\,Air} - \dot{q}_{rad\,Bar} - \dot{q}_{conv}$$

$$\dot{q}_{cond} = k A_c \frac{(T_B - T_T)}{\Delta x}$$

$$\dot{q}_{conv} = h A_s (T_T - T_g)$$

$$\dot{q}_{rad\,Air} = \alpha \sigma A_s T_g^4$$

$$\dot{q}_{rad\,Bar} = \varepsilon \sigma A_s T_T^4$$

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A 8
C 8

Set #11 Section 4.4 #13, 17, 27, 33, 43, 53, 101

13. $f(x) = -2x^3 + 6x^2 - 3$

$f'(x) = -6x^2 + 12x$

$f''(x) = -12x + 12$

C.P. $\Rightarrow 0 = -6x^2 + 12x = x(-6x+12) = \boxed{0, 2}$

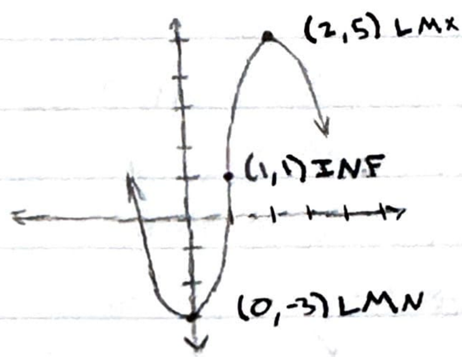
$f''(0) = -12(0) + 12 = 12$, $\boxed{\text{Concave Up}}$

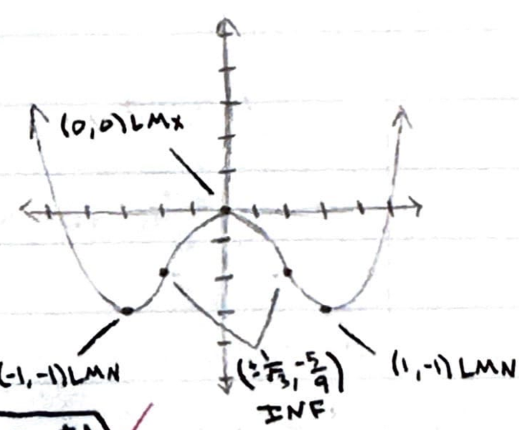
$f''(2) = -12(2) + 12 = \boxed{-12}$, $\boxed{\text{Concave Down}}$

INF $\Rightarrow 0 = -12x + 12 = \boxed{1} \Rightarrow f(1) = 1$, $\boxed{(1,1)}$

$f(0) = -2(0)^3 + 6(0)^2 - 3 = -3$, $\boxed{\text{LMini } (0,-3)}$

$f(2) = -2(2)^3 + 6(2)^2 - 3 = 5$, $\boxed{\text{LMaxi } (2,5)}$





17. $f(x) = x^4 - 2x^2 = x^2(x^2 - 2)$

4

$f'(x) = 4x^3 - 4x$ ✓

$f''(x) = 12x^2 - 4$ ✓

C.P. $\Rightarrow 0 = 4x^3 - 4x = 4x(x+1)(x-1) = \boxed{0, \pm 1}$ ✓

$f''(0) = 12(0)^2 - 4 = -4$, $\boxed{\text{Concave Down}}$ ✓

$f''(-1) = 12(-1)^2 - 4 = 8$, $\boxed{\text{Concave Up}}$ ✓

$f''(1) = 12(1)^2 - 4 = 8$, $\boxed{\text{Concave Up}}$ ✓

INF $\Rightarrow 0 = 12x^2 - 4 = \sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}} \Rightarrow f(\pm\frac{1}{\sqrt{3}}) = -\frac{5}{9} \Rightarrow \boxed{(\pm\frac{1}{\sqrt{3}}, -\frac{5}{9})}$ ✓

$f(0) = 0$, $\boxed{\text{LMX: } (0,0)}$ $f(\pm 1) = -1$, $\boxed{\text{LMN: } (\pm 1, -1)}$ ✓

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27. $f(x) = \sin x \cos x\ ,\quad 0 \le x < \pi$

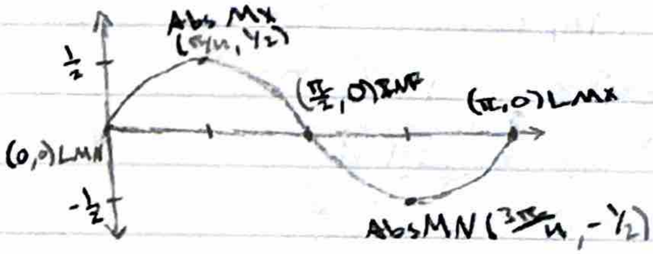
$f'(x) = -\sin^2 x + \cos^2 x = \cos 2x$

$f''(x) = -2\sin 2x$

C.P. $\Rightarrow 0 = \cos 2x = \boxed{\frac{\pi}{4}, \frac{3\pi}{4}}$

$f''(\frac{\pi}{4}) = \boxed{\text{C Down}}$ $\quad f''(\frac{3\pi}{4}) = \boxed{\text{C Up}}$

INF $\Rightarrow 0 = -2\sin 2x = \boxed{0, \frac{\pi}{2}, \frac{\pi}{0}}$



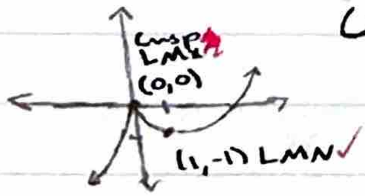
33. $f(x) = 2x - 3x^{2/3}$ ✓ $\quad$ Rising: $(-\infty, 0)$ and $(1, \infty)$

4

$f'(x) = 2 - 2x^{-\frac{1}{3}}$ ✓ $\quad$ Falling: $(1, \infty)$

$f''(x) = \frac{2}{3}x^{-\frac{4}{3}}$ ✓ $\quad$ LMX: (0,0) $\quad$ LMN: (1,−1)

CUp: $(-\infty, 0)$ and $(0, \infty)$ ✓ $\quad$ NO INF But Cusp: $x=0$



43. $f(x) = \frac{8x}{x^2+4}$ $\quad$ Rising: $(-2, 2)$

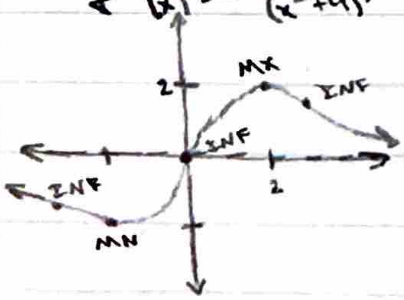
$f'(x) = \frac{-8(x^2-4)}{(x^2+4)^2}$ $\quad$ Falling: $(-\infty, 2)$ and $(2, \infty)$

$f''(x) = \frac{16x(x^2-12)}{(x^2+4)^3}$ $\quad$ L/A MX: (2,2) $\quad$ L/A MN: (−2,2)

CUp: $(-2\sqrt{3}, 0)$ and $(2\sqrt{3}, \infty)$

CDown: $(0, 2\sqrt{3})$ and $(-\infty, -2\sqrt{3})$

INF: $(\pm 2\sqrt{3}, \pm\sqrt{3})$ and $(0,0)$



$y=0$ horizontal asymptote

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Math1411-17
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Set #11 Section 4.4 #13,17,27,33,43,53,101 Continued

53. $y' = x(x^2-12) = x(x-2\sqrt{3})(x+2\sqrt{3})$

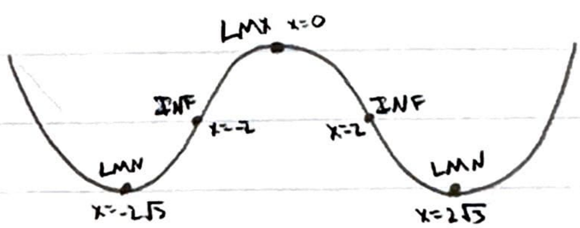
$y' = ---\,|_{-2\sqrt{3}}\,+++\,|_{0}\,---\,|_{2\sqrt{3}}\,+++$

LMX: $x=0$ LMN: $\pm 2\sqrt{3}$

$y'' = 1(x^2-12) + x(2x) = 3(x-2)(x+2)$

$y'' = +++\,|_{-2}\,---\,|_{2}\,+++$

INF: $x \pm 2$



101. $y' = (x-1)^2(x-2)$

$y'' = 2(x-1)(x-2) + (x-1)^2$

$y' = ---\,|_{2}\,+++$

LMN: $x=2$ LMX: None

$y'' = +++\,|_{1}\,---\,|_{\frac{5}{3}}\,+++$

INF: $x = 1$ or $\frac{5}{3}$

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A 8
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Set #4 - Sections 2.1 + 3.1 + 3.2

2.1

1. a. $\frac{\Delta f}{\Delta x} = \frac{f(3)-f(2)}{3-2} = \frac{28-9}{1} = \boxed{19}$

   b. $\frac{\Delta f}{\Delta x} = \frac{f(1)-f(-1)}{1-(-1)} = \frac{2-0}{2} = \boxed{1}$

9. $\frac{\Delta y}{\Delta x} = \frac{((2+h)^2-2(2+h)-3)-(2^2-2(2)-3)}{h} = \frac{4+4h+h^2-4-2h-3-(-3)}{h} = \frac{2h+h^2}{h} = 2+h$

   As $h \to 0, 2+h \to 2 \Rightarrow$ at $P(2,-3)$ the slope is 2.

   b. $y-(-3) = 2(x-2) \Rightarrow y+3 = 2x-4 \Rightarrow \boxed{y=2x-7}$

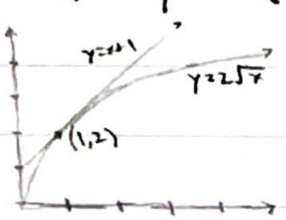
21. a. $[0,1]: \frac{15-0}{1-0} = \boxed{15\text{ mph}}$, $[1,2.5]: \frac{20-15}{2.5-1} = \boxed{\frac{10}{3}\text{ mph}}$, $[2.5,3.5]: \frac{30-20}{3.5-2.5} = \boxed{10\text{ mph}}$

    b. $t=\frac{1}{2}: \frac{15-7.5}{1-0.5} = \boxed{15\text{ mph}}$ $t=2: \frac{20-20}{2.5-2} = \boxed{0\text{ mph}}$ $t=3: \frac{30-21}{3.5-3} = \boxed{18\text{ mph}}$

3.1

7. $m = \lim_{h \to 0} \frac{2\sqrt{1+h}-2\sqrt{1}}{h} = \lim_{h \to 0} \frac{2\sqrt{1+h}-2}{h} \cdot \frac{2\sqrt{1+h}+2}{2\sqrt{1+h}+2} = \lim_{h \to 0} \frac{4(1+h)-4}{2h(\sqrt{1+h}+1)} = \lim_{h \to 0} \frac{2}{\sqrt{1+h}+1} = 1$

   at $(1,2)$: $y = 2+1(x-1) \Rightarrow \boxed{y = x+1}$

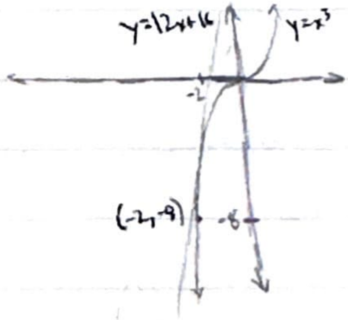


9. $m = \lim_{h \to 0} \frac{(-2+h)^3-(-2)^3}{h} = \lim_{h \to 0} \frac{-8+12h-6h^2+h^3+8}{h}$

   $= \lim_{h \to 0} (12-6h+h^2) = 12$

   at $(-2,-8)$: $y = -8+12(x-(-2)) \Rightarrow$

   $\boxed{y = 12x+16}$



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4

11. $m = \lim_{h\to0} \frac{[(2+h)^2+1]-5}{h} = \lim_{h\to0} \frac{(5+4h+h^2)-5}{h} = \lim_{h\to0} \frac{h(4+h)}{h} = \boxed{4}$

at $(2,5)$: $\boxed{y-5=4(x-2)}$

21. At $x=3$, $y=\frac{1}{2} \Rightarrow m = \lim_{h\to0} \frac{\frac{1}{(3+h)-1}-\frac{1}{2}}{h} = \lim_{h\to0} \frac{2-(2+h)}{2h(2+h)} = \lim_{h\to0} \frac{-h}{2h(2+h)} = \boxed{-\frac{1}{4}}$

23. $m=0 \Rightarrow \lim_{h\to0} \frac{[(x+h)^2+4(x+h)-1]-(x^2+4x-1)}{h} = \lim_{h\to0} \frac{(x^2+2xh+h^2+4x+4h-1)-(x^2+4x-1)}{h} =$

$\lim_{h\to0} \frac{(2xh+h^2+4h)}{h} = \lim_{h\to0}(2x+h+4) = 2x+4 \Rightarrow 2x+4=0 \ \boxed{(-2,-5)}$

3.2

7. $\lim_{h\to0} \frac{2(x+h)^3-2x^3}{h} = \lim_{h\to0} \frac{2(x^3+3x^2h+3xh^2+h^3)-2x^3}{h} = \lim_{h\to0} \frac{6x^2h+6xh^2+2h^3}{h} =$

$\lim_{h\to0}(6x^2+6xh+2h^2) = \boxed{6x^2}$

4

9. $\lim_{h\to0} \frac{\left(\frac{t+h}{2(t+h)+1}\right)-\left(\frac{t}{2t+1}\right)}{h} = \lim_{h\to0} \frac{(t+h)(2t+1)-t(2t+2h+1)}{(2t+2h+1)(2t+1)h} = \lim_{h\to0} \frac{h}{(2t+2h+1)(2t+1)h} =$

$\frac{1}{(2t+1)(2t+1)} = \boxed{\frac{1}{(2t+1)^2}}$

13. $f(x)=x+\frac{9}{x}$ and $f(x+h)=(x+h)+\frac{9}{(x+h)} \Rightarrow \frac{f(x+h)-f(x)}{h} = \frac{\left[(x+h)+\frac{9}{(x+h)}\right]-\left[x+\frac{9}{x}\right]}{h} =$

$\frac{x(x+h)^2+9x-x^2(x+h)-9(x+h)}{x(x+h)h} = \boxed{\frac{x^2+xh-9}{x(x+h)}}$; $f'(x) = \lim_{h\to0} \frac{x^2+xh-9}{x(x+h)} = \frac{x^2-9}{x^2} = 1-\frac{9}{x^2}$; $m=f'(-3)=\boxed{0}$

17. $f'(x) = \lim_{h\to0} \frac{-8}{\sqrt{x+h-2}\sqrt{x-2}(\sqrt{x-2}+\sqrt{x+h-2})} = \frac{-8}{\sqrt{x-2}\sqrt{x-2}(\sqrt{x-2}+\sqrt{x-2})} = \boxed{\frac{-4}{(x-2)\sqrt{x-2}}}$; $m=f'(6)=$

$\frac{-4}{4\sqrt{4}} = -\frac{1}{2} \Rightarrow y-4=-\frac{1}{2}(x-6) \Rightarrow \boxed{y=-\frac{1}{2}x+7}$

43. a. The function is differentiable on its domain $-3\le x\le 2$ (smooth)

b. none

c. none

Mohammad Elassaad-Lamp
Math 141-12
10/5/18

Set #4 - Sections 2.1 + 3.1 + 3.2 (Continued)

3.2 (continued)

45. a. The function is differentiable on $-3 \le x < 0$ and $0 < x \le 3$

b. none

c. The function is neither at $x=0$ since $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

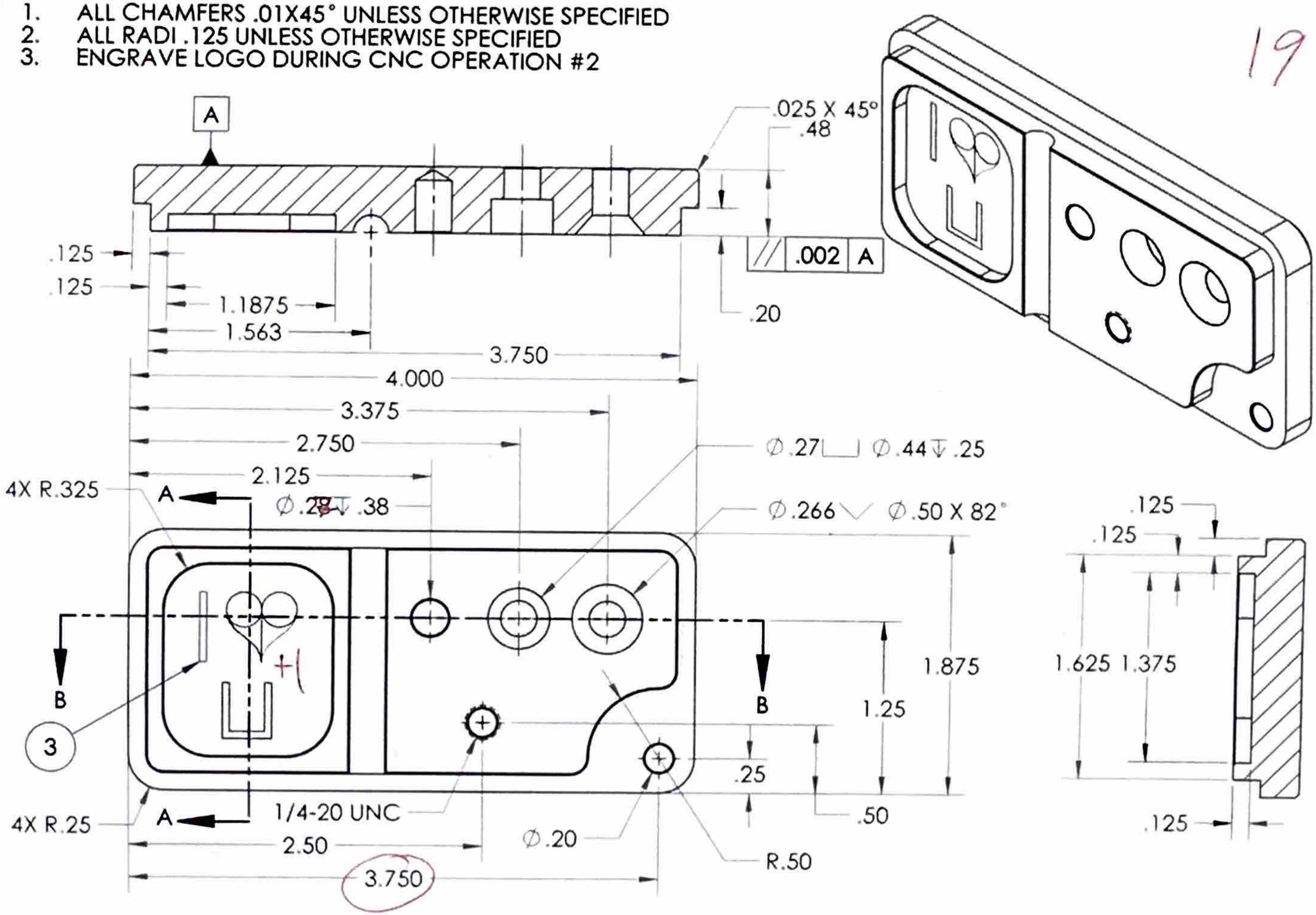
47. a. f is differentiable on $-1 \le x < 0$ and $0 < x \le 2$

b. f is continuous but not differentiable at $x=0$: $\lim_{x \to 0} f(x) = 0$ exists but there is a cusp at $x=0$ so $f'(0) = \lim_{h \to 0} \frac{f(0+h)-f(0)}{h}$ does not exist

c. none

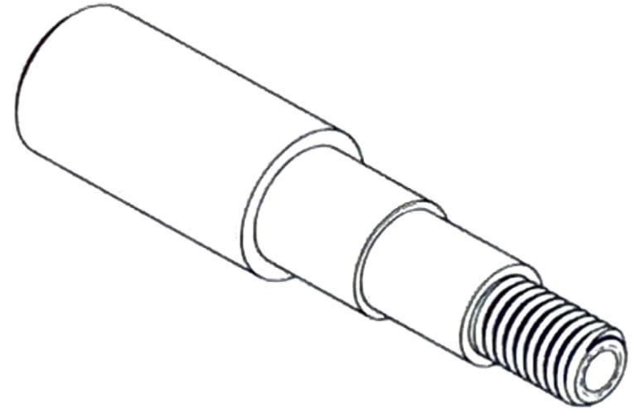
19

NOTES:
1. ALL CHAMFERS .01X45° UNLESS OTHERWISE SPECIFIED
2. ALL RADI .125 UNLESS OTHERWISE SPECIFIED
3. ENGRAVE LOGO DURING CNC OPERATION #2

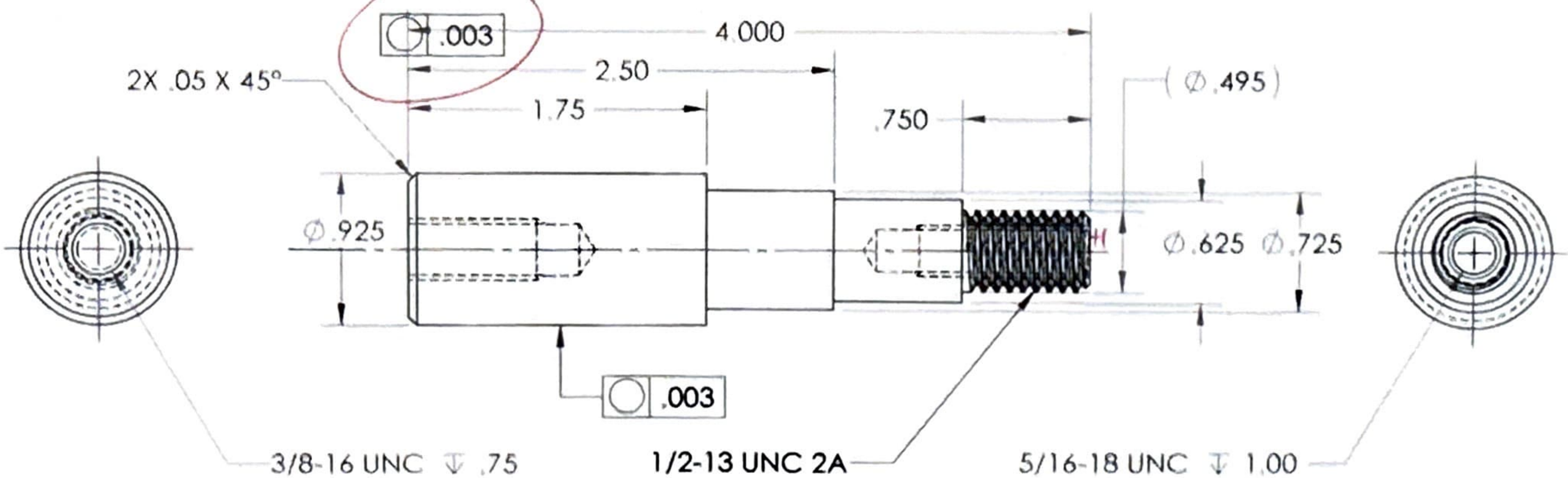


UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN INCHES TOLERANCES: ONE PLACE DECIMAL ± .1 TWO PLACE DECIMAL ± .01 THREE PLACE DECIMAL ± .005	INTERPRET DRAWING PER ASME Y14.5 2009	CAL POLY Manufacturing Engineering	MATERIAL: 6061-T6 ALUMINUM	TITLE: IME 144 MILL PROJECT #1 Lab 6
		DATE: 10/2/18	DRAWN BY: MOHAMMAD S ELASSAAD	SHEET 1 OF 1 · SCALE: 1:1 · REV B · SIZE A

NOTES:
1. CHAMFER / DEBURR SHARP EDGES
2. DRILL 1.00"↧ FOR 3/8-16 TAP
3. DRILL 1.25"↧ FOR 5/16-18 TAP



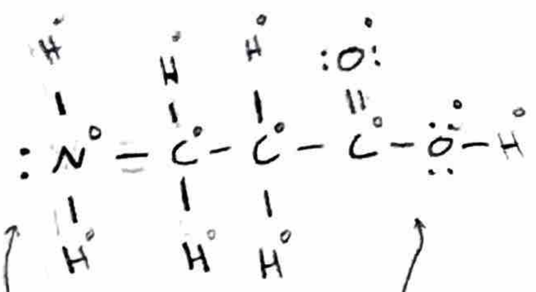
20



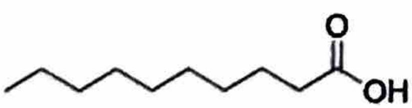
UNLESS OTHERWISE SPECIFIED: DIMENSIONS ARE IN INCHES TOLERANCES: ONE PLACE DECIMAL ± .1 TWO PLACE DECIMAL ± .01 THREE PLACE DECIMAL ± .005	INTERPRET DRAWING PER ASME Y14.5 2009	CAL POLY Manufacturing Engineering DATE: 10/2/18	MATERIAL: 6061-T6 ALUMINUM DRAWN BY: MOHAMMAD ELASSAAD	TITLE: IME 144 LATHE PROJECT #1 Lab 6 SHEET 1 OF 1	SCALE: 1:1	REV B	SIZE A

Short Answer and Calculations *Show all your work where appropriate in order to receive full or partial credit!* **Remember to include units and watch significant figures on calculations!**

23. (16 pts) Draw a Lewis Structure with all closed-shell atoms and minimized formal charges on atoms for the following molecule, making sure you include all lone pair electrons. DO show the formal charges on the Lewis Structure. Also show ALL the atoms, even H bonded to C. Assume no resonance structures are possible. Fill in the rest of the table (on the right). Then answer all the questions below the table.

$NH_2CH_2CH_2COOH$ 6-4-2=0 1 2 3 **3-aminopropanoic acid** Formal charge is 0 Total # of electrons is 36  5-2-3=0 4-4=0 6-4-2=0	**Name the molecular geometry of the nitrogen in this molecule:** Trigonal Pyramidal **Name the molecular geometry of carbon #3:** Trigonal Planar **Is this molecule polar?** Yes, hydrogen bonds + lone pairs + diff. bonds

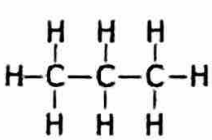
Would you PREDICT the molecule 3-aminopropanoic acid (above) to have a higher or lower boiling point than decanoic acid, shown below? Explain your answer concisely. +1/2



great 3-aminopropanoic acid has a lower boiling point than decanoic acid. They both have hydrogen bonds because they are acids and 3-aminopropanoic acid also has hydrogen bonds to nitrogen. They are both polar. However, decanoic acid has a longer hydrocarbon chain which means it has more dispersion forces because of higher surface area.

This means that decanoic acid has a greater total of IMFs and a higher boiling point.

Would you PREDICT the molecule 3-aminopropanoic acid (aboe) to have a higher or lower boiling point than propane, shown below? Explain your answer concisely.



3-aminopropanic acid has a higher boiling point than propane because it has more IMFs. Although they have the same hydrocarbon chain length, 3-aminopropanoic acid has hydrogen bonds and is polar which adds to its IMFs and means its bonds are stronger therefore it has a higher boiling point.

I don't want an explanation so long it takes up all this space!!!!

16.5

24. (20 pts) A researcher at the photovoltaics company you work for measured the band gap of three compound semiconductors. Those three semiconductors were indium arsenide (InAs), indium phosphide (InP), and aluminum phosphide (AlP). However, the researcher did not record which band gap corresponded to which semiconductor, and now he's worried he's going to be fired. Given the three band gaps listed below, help this guy out and assign each band gap to the appropriate semiconductor and list those in the table in the third column. In order to receive credit, you must explain your reasoning ***briefly but completely*** below the table.

Trial	Band Gap (eV)	Which semiconductor?
1	2.45	AlP
2	1.35	InP
3	0.36	InAs

✓

good

Explanation:

Out of the three compounds, AlP has elements with the highest electronegativity, electron affinity, and ionization energy. It is also made of smaller elements so it has more orbital overlap and stronger bonds. It is also more tightly packed. All of this means it requires more energy to move electrons between conduction band and valence band which means a larger bandgap. InAs is the opposite and InP is in the middle.

b. Let's say you want to use one of these materials in a solar panel, to convert solar energy to electrical energy. If the peak solar radiation is composed of photons with wavelengths only in the visible part of the spectrum, which material will you be able to use? *Refer to the materials using the band gaps, not the chemical formulas, in case you messed up in part a).* You must show mathematical work to receive credit. ✓

1) $2.45\,eV \times \frac{1.602\times10^{-19}\,J}{1\,eV} = 3.9249\times10^{-19}\,J = \frac{hc}{\lambda} = \lambda = \boxed{506\,nm}$ this is in visible light range so 2.45 eV

2) $1.35\,eV \times \frac{1.602\times10^{-19}\,J}{1\,eV} = 2.1627\times10^{-19}\,J = \frac{hc}{\lambda} = \lambda = 919\,nm$

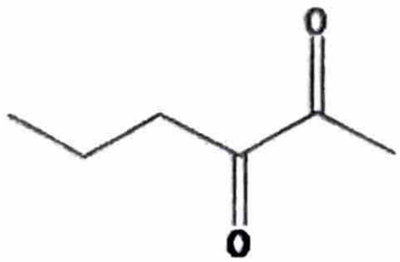
3) $0.36\,eV \times \frac{1.602\times10^{-19}\,J}{1\,eV} = 5.7672\times10^{-20}\,J = \frac{hc}{\lambda} = \lambda = 3446\,nm$

c. Now let's say you want to use the indium arsenide material as a basis for a new series of LEDs. What element would you substitute for the indium in order to create an LED that emits light with lower wavelength than the pure indium arsenide? Explain your choice briefly. *You don't need to know the exact band gap for the indium arsenide to answer this!*

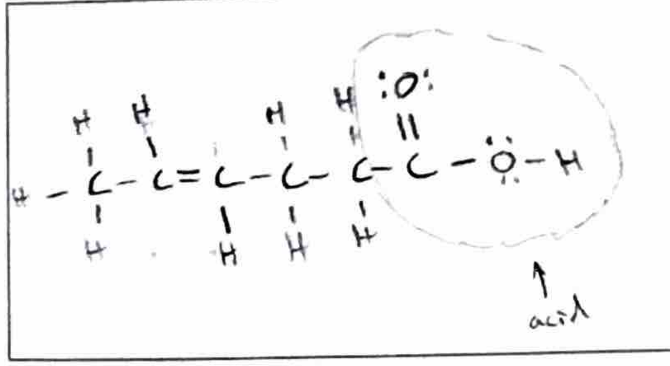
20

I would choose Ga. Ga is in the same column so it can be substituted. It has higher EA, EN, IE, so it has stronger + tighter bonds so it has larger band band gap energy because it takes more energy to move electrons. More energy means less wavelength therefore 7 it will emit the proper color of light (proper wavelength).

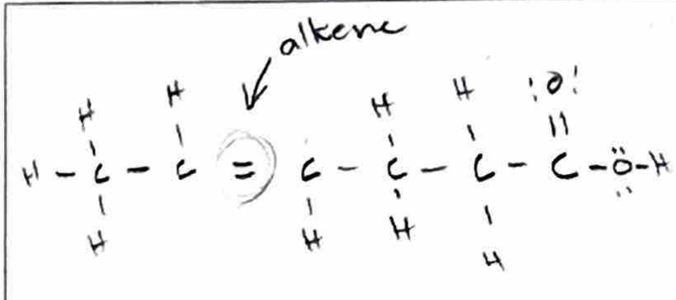
25. (12 pts) 2,3-hexandione ($C_6H_{10}O_2$), shown below, is an organic molecule with an odor described as buttery, creamy, oily, toasted and like caramel.§ It is a FDA approved food additive. It is a constituent, oddly enough, of coffee, peach, roast chicken, beer, shoyu and clam.* Draw the following isomers of this molecule.



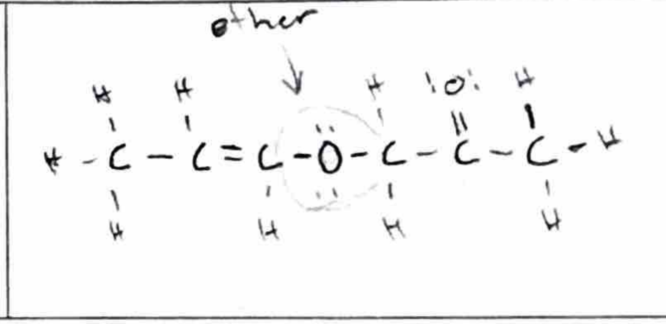
Draw an isomer containing a carboxylic acid



Draw an isomer containing an alkene



Draw an isomer containing an ether



12

§ http://www.thegoodscentscompany.com/data/rw1014711.html
*https://pubchem.ncbi.nlm.nih.gov/compound/2_3-hexanedione

26. (8 pts) Balance each of the following reactions and write the net ionic equations under each reaction:

a. 2 NaCl (aq) + 1 Pb(NO_3)$_2$ (aq) → 2 $NaNO_3$ (aq) + 1 $PbCl_2$ (s)   Precipitation

$\cancel{2Na^{+1}_{(aq)}} + 2Cl^{-1}_{(aq)} + Pb^{+2}_{(aq)} + \cancel{2(NO_3)^{-1}_{(aq)}} \rightarrow \cancel{2Na^{+1}_{(aq)}} + \cancel{2(NO_3)^{-1}_{(aq)}} + PbCl_2(s)$

$\boxed{2Cl^{-1}(aq) + Pb^{+2}(aq) \rightarrow Pb^{+2}Cl^{-1}_2(s)}$ ✓

b. 2 Al (s) + 3 H_2SO_4 (aq) → 1 $Al_2(SO_4)_3$ (aq) + 3 H_2 (g)   Redox

$2Al^{0}(s) + 6H^{+1}(aq) + \cancel{3(SO_4)^{-2}(aq)} \rightarrow Al^{+3}_{2}(aq) + \cancel{3(SO_4)^{-2}(aq)} + 3H^{0}_2(g)$

$\boxed{2Al^{0}(s) + 6H^{+1}(aq) \rightarrow Al^{+3}_2(aq) + 3H^{0}_2(g)}$ 2Al+3 − 1/2

7.5

c. 3 NaOH (aq) + 1 H_3PO_4 (aq) → Na_3PO_4 (aq) + 3 H_2O (l)   Acid-Base

$\cancel{3Na^{+1}(aq)} + 3(OH)^{-1}(aq) + 3H^{+1}(aq) + \cancel{(PO_4)^{-3}(aq)} \rightarrow \cancel{3Na^{+1}(aq)} + \cancel{(PO_4)^{-3}(aq)} + 3H^{+1}_2O^{-2}(l)$

$\boxed{3(OH)^{-1}(aq) + 3H^{+1}(aq) \rightarrow 3H^{+1}_2O^{-2}(l)}$

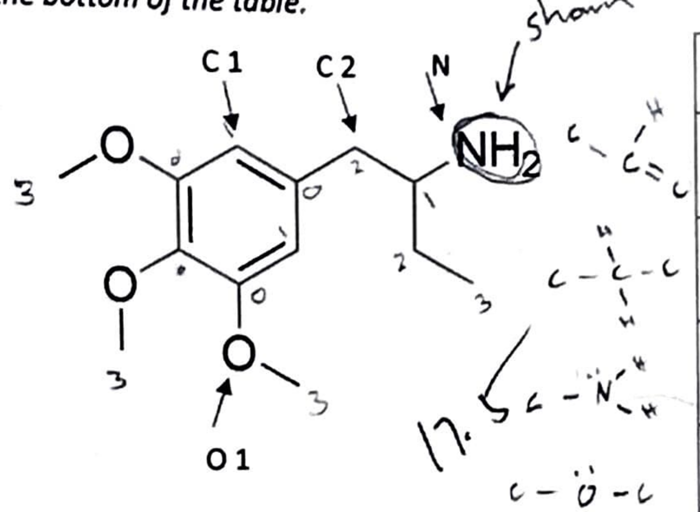
d. 1 $MgCO_3$ (s) + 2 HCl (aq) → 1 $MgCl_2$ (aq) + 1 CO_2 (g) + 1 H_2O (l)   Acid Base

$Mg^{+2}(CO_3)^{-2}(s) + 2H^{+1}(aq) + \cancel{2Cl^{-1}(aq)} \rightarrow Mg^{+2}(aq) + \cancel{2Cl^{-1}(aq)} + C^{+4}O^{-2}_2(g) + H^{+1}_2O^{-2}(l)$

$\boxed{Mg^{+2}(CO_3)^{-2}(s) + 2H^{+1}(aq) \rightarrow Mg^{+2}(aq) + C^{+4}O^{-2}_2(g) + H^{+1}_2O^{-2}(l)}$

27. (18 pts) Mescaline is a naturally occurring psychedelic alkaloid, found in the peyote cactus. Its hallucinogenic effects are comparable to those of LSD and psilocybin, the psychedelic found in some mushrooms.* The structure of mescaline is shown below.

For the indicated atoms, identify the electronic geometry and molecular geometry. Then answer the questions at the bottom of the table.



Remember this is NOT a Lewis Structure, it's a line diagram, so information is missing here!

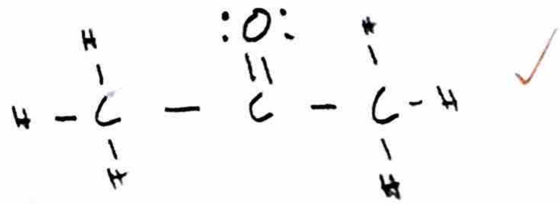
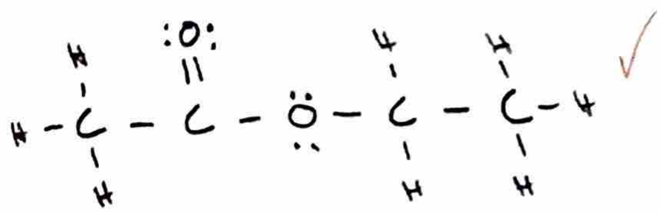
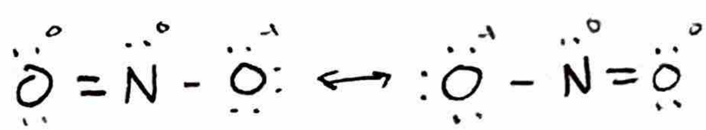
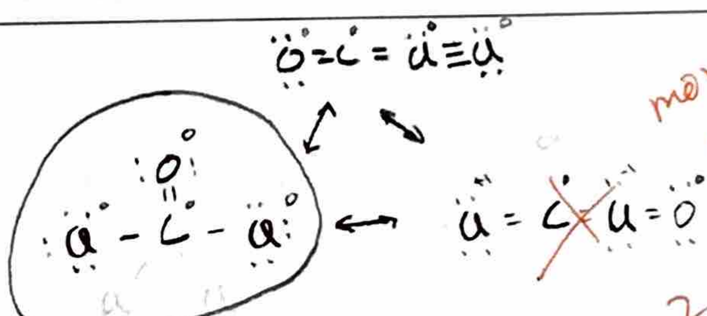
3+3+3+1+1+2+1+2+3-2=17

Atom	Electronic Geometry	Molecular Geometry
C 1	Trigonal Planar	Trigonal Planar
C 2	Tetrahedral	Tetrahedral
N	Tetrahedral	Trigonal Pyramidal
O 1	Tetrahedral	Bent
How many H atoms are *not shown* in this structure?	17 H atoms not shown	19

1/2

*Mescaline is a Schedule I controlled substance and so even though its thought to have some therapeutic uses, its availability to researchers is very limited. https://en.wikipedia.org/wiki/Mescaline

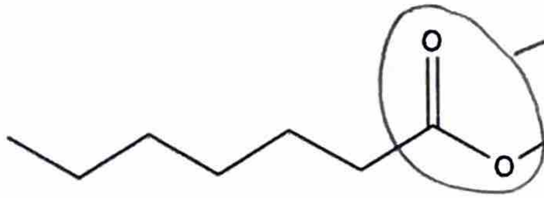
1. (12 pts) Draw a good Lewis Structure for each of the following molecules or ions. *Include all lone pair electrons.* **Include formal charges on c) & d), but not on a) and b).** *Resonance structures are possible for c) and d) as well, so draw them!*

Formula	Lewis Structure (s)
a) CH_3COCH_3 Acetone, a ketone with the systematic name of propanone, a common organic solvent that is miscible with water, produced and disposed of naturally in the human body	
b) $CH_3COOCH_2CH_3$ Ethyl acetate, an ester A common organic solvent, used in glues, nail polish removers and cigarettes.	
# of electrons = 6+6+5+1=18 Nitrite ion, used in food production industry to cure meats c) NO_2^- There are resonance structures are possible here so draw them, include formal charges too.	 Both have formal charges of -1
# of e = 4+6+7+7=24 Phosgene, a colorless gas used as a chemical weapon in WWI d) $COCl_2$ (carbon is central) There are resonance structures possible here, **draw at least two structures** total, include formal charges, and **indicate** which of two structures is better by **circling it.**	 move e⁻ not atoms -2

10

2. (4 pts) Identify each of the oxygen-containing functional groups circled in each molecule. Write the name of the functional group next to or below the molecule.

a.

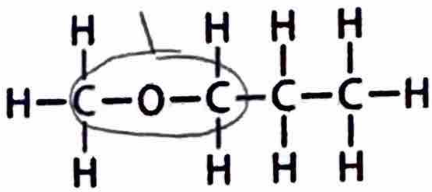


Ester

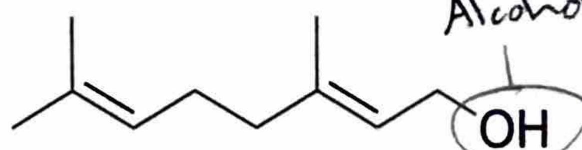
3

b.

Ether

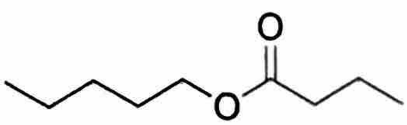


c.

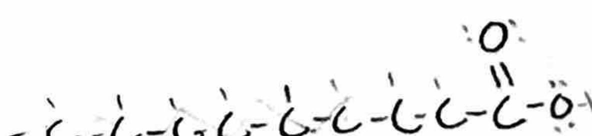


Alcohol

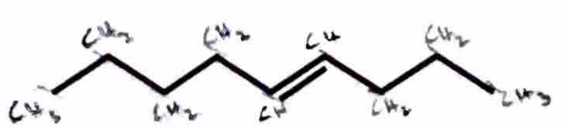
6 3. Pentyl Butyrate ($C_9H_{18}O_2$), shown below, is an organic molecule with a smell similar to pears, and oddly enough, it is used as an additive in cigarettes.* Draw the following indicated isomers of this molecule. You can draw line diagrams or structural formulas.



6

Draw an isomer containing an alcohol group	Draw an isomer containing a carboxylic acid group
Alcohol Group 	

4. (2 pts) Write out the correct molecular and condensed formulas for the following line diagram.



3

Condensed: $CH_3\ CH_2\ CH_2\ CH_2\ CH\ CH\ CH_2\ CH_2\ CH_3$

Molecular: C_9H_{18}

molar mass = g/mol

Molarity (M) = $\frac{\text{moles}}{\text{liter}}$

(10 pts) 1. Ethyl acetate is the solvent in many fingernail polish removers and is used to decaffeinate coffee beans and tea leaves. It is prepared by reacting ethanol with acetic acid as shown below.

ethanol acetic acid ethyl acetate water

$$CH_3CH_2OH\ (l) + CH_3COOH\ (aq) \rightarrow CH_3COOCH_2CH_3\ (aq) + H_2O\ (l)$$

46.069 g/mol 60.052 g/mol 88.106 g/mol 18.015 g/mol

a) If 150. mL of ethanol (0.789 g/mL) and 122. mL of 15.0 M acetic acid are reacted, which reactant is limiting this reaction? Work smart here – that is, do the minimum work necessary.

Ethanol = $\frac{150\text{ mL}}{1} \times \frac{0.789\text{ g}}{\text{mL}} = \frac{118.35\text{ g } CH_3CH_2OH}{1}$

$\frac{118.35\text{ g } CH_3CH_2OH}{1} \times \frac{1\text{ mol}}{46.069\text{ g}} \times \frac{1\text{ mol } CH_3COOCH_2CH_3}{1\text{ mol } CH_3CH_2OH} \times \frac{88.106\text{ g}}{\text{mol}} = \boxed{226\text{ g Ethyl Acetate}}$

Acetic Acid = $\frac{122\text{ mL}}{1} \times \frac{1\text{ L}}{1000\text{ mL}} \times \frac{15.0\text{ moles}}{1\text{ L}} = \frac{1.83\text{ moles}}{1} \times \frac{60.052\text{ g}}{\text{mol}} = \frac{109.895\text{ g } CH_3COOH}{1}$

$\frac{109.895\text{ g } CH_3COOH}{1} \times \frac{1\text{ mol}}{60.052\text{ g}} \times \frac{1\text{ mol } CH_3COOCH_2CH_3}{1\text{ mol } CH_3COOH} \times \frac{88.106\text{ g}}{\text{mol}} = \boxed{161\text{ g Ethyl Acetate}}$

161 g < 226 $\Rightarrow$ $\boxed{\text{Acetic Acid is the limiting reagent}}$

b) How many grams of *ethyl acetate* would be formed? Assume complete reaction. Show all your work and circle your answer.

Acetic Acid = $\frac{122\text{ mL}}{1} \times \frac{1\text{ L}}{1000\text{ mL}} \times \frac{15.0\text{ moles}}{1\text{ L}} = \frac{1.83\text{ moles}}{1} \times \frac{60.052\text{ g}}{\text{mol}} = 109.895\text{ g } CH_3COOH$

$\frac{109.895\text{ g } CH_3COOH}{1} \times \frac{1\text{ mol}}{60.052\text{ g}} \times \frac{1\text{ mol Ethyl Acetate}}{1\text{ mol Acetic Acid}} \times \frac{88.106\text{ g}}{\text{mol}} = \boxed{161\text{ g Ethyl Acetate}}$

(Theoretical Yield)

c) What would the concentration of the resulting solution of ethyl acetate be? Assume the volumes of the reactants are additive to give the final volume of solution. Show all your work and circle your answer.

concentration = $\frac{\text{mass ethyl acetate}}{\text{mass of solution}}$

$\frac{\text{mol}}{\text{L}}$

Mass of solution = ethyl acetate + water + excess reagent

−2

BUT

Therefore, mass of solution = Ethanol + Acetic Acid

$\Rightarrow \frac{161\text{ g Ethyl Acetate}}{118.35\text{ g Ethanol} + 109.895\text{ g Acetic Acid}} =$ The concentration of Ethyl Acetate is 0.705 or 70.5%

272 mL

8

2. (10 pts) Quinine is a naturally occurring alkaloid found in the bark of the cinchona tree that has a long history as a medicinal compound. It was (I think) the first successful treatment for malaria. While it is not recommended at the go-to treatment for that anymore, it is still used medicinally today to treat lupus and arthritis, and it has been used in the past to treat everything from restless leg syndrome to diarrhea to shivering. It is also the ingredient in tonic water responsible for the bitter taste. It was first isolated in a lab from a cinchona tree in 1820, but extracts from the bark have been used to treat malaria as far back as 1631.* (And yes, absolutely none of that is necessary for working this problem – I just find it fascinating so I wrote it all down here.)

C H O N

a) Quinine contains only carbon, hydrogen, oxygen and nitrogen. A 0.487 g sample that is thought to be quinine was combusted in oxygen and found to produce 1.321 g of carbon dioxide, 0.325 g of water, and 0.0421 g of nitrogen. Find the empirical formula for quinine. Show all your work explicitly and circle your answer.

$C_{\square}H_{\square}O_{\square}N_{\square} + O_2 \rightarrow CO_2 + H_2O + N$

Carbon $= \frac{1.321\,g\,CO_2}{1} \times \frac{12.011\,g\,C}{44.009\,g\,CO_2} = \frac{0.3605\,g\,C}{1} \times \frac{1\,mol\,C}{12.011\,g\,C} = 0.030014154\,mol\,C$

Hydrogen $= \frac{0.325\,g\,H_2O}{1} \times \frac{2.016\,g\,H}{18.015\,g\,H_2O} = \frac{0.0364\,g\,H}{1} \times \frac{1\,mol\,H}{1.008\,g\,H} = 0.03611\,mol\,H$

Nitrogen $= \frac{0.0421\,g\,N}{1} \times \frac{1\,mol\,N}{14.007\,g\,N} = 0.0030564\,mol\,N$

Oxygen $= 0.487\,g - (0.3605\,g\,C + 0.0364\,g\,H + 0.0421\,g\,N) = 0.0480\,g\,O$

$0.0480\,g\,O \times \frac{1\,mol}{15.999\,g\,O} = 0.003000188\,mol\,O$

carbon $= \frac{0.030}{0.003} = 10$

Hydrogen $= \frac{0.036}{0.003} = 12$

Nitrogen $= \frac{0.003}{0.003} = 1$

Oxygen $= \frac{0.003}{0.003} = 1$

$\Rightarrow$ $\boxed{\text{Emperical Formula: } C_{10}H_{12}O_1N_1}$

b) By a separate method, the molecular weight of this compound was found to be 324 g/mol. Determine the molecular formula for quinine. Show all your work explicitly and circle your answer.

C = 10 mol C $\frac{12.011\,g\,C}{1\,mol\,C} = 120.11\,g\,C$

H = 12 mol H $\times \frac{1.008\,g\,H}{1\,mol\,H} = 12.096\,g\,H$

O = 1 mol O $\times \frac{15.999}{mol} = 15.999\,g\,O$

N = 1 mol N $\times \frac{14.007}{mol} = 14.007\,g\,N$ +

162.212 g = emperical weight g/mol

$\Rightarrow \frac{324\,g/mol}{162.2129/mol} = 2.00$ = factor of

$\Rightarrow$ multiply emperical formula by factor of 2

$\Rightarrow \boxed{C_{20}H_{24}O_2N_2}$

* https://en.wikipedia.org/wiki/Quinine. Also there's a fascinating book about quinine that I read several years ago: "Quinine: Malaria and the Quest for a Cure That Changed the World" by Flammetta Rocco

10

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Calculus 3 Midterm 1 Review

$\boxed{10.1}$ Sequences (Memorize Famous Limits)

$\{a_n\}$ = sequence with commas

$\lim_{n\to\infty} a_n = L$ - if L = real # then point of convergence
- if L = not real then divergence

If a function is monotonic and is bounded then it is convergent.
monotonic → increasing or decreasing

$\boxed{10.2}$ Infinite Series

The series is the sequence of partial sums

$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 \ldots$ $\{S_n\}$: $S_1 = a_1$
$S_2 = a_1 + a_2$
$S_3 = a_1 + a_2 + a_3$

Series converges if S_n converges

$\sum_{n=1}^{\infty} a_n = \{S_n\}$ and $\lim_{n\to\infty} S_n = L \Rightarrow \sum_{n=1}^{\infty} a_n = L$

Sum of the series is the $\lim_{n\to\infty} S_n$

Geometric Series $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 \ldots$

$\lim_{n\to\infty} r^n = 0$ if $|r|<1$ then $\lim_{n\to\infty} S_n = \frac{a}{1-r}$ (converges)

$\frac{a}{1-r}$ = sum of the series, also where it converges

if $|r| > 1$ then series diverges

if $|r| = 1$ then diverges

Theorem: If $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$ $p \Rightarrow q$

False: If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ is convergent $q \not\Rightarrow p$

Ex: Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

N^{th} Term Divergence Test (Sequence)

If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ will diverge

P-Series Test

$\sum_{n=1}^{\infty} \frac{1}{x^p}$ converges if and only if $p > 1$

$\boxed{10.3}$ Integral Test (good for ln)

$f(x)$ is positive, continuous, and decreasing on $[k, \infty)$

- If $\int_k^\infty f(x)dx$ is convergent, then $\sum_{n=k}^{\infty} f(n)$ is also convergent
- If $\int_k^\infty f(x)dx$ is divergent, then $\sum_{n=k}^{\infty} f(n)$ is also divergent

$f(x) = \frac{1}{x}$ over $[1,\infty)$, $\int_1^\infty \frac{1}{x}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}dx = \lim_{t\to\infty}[\ln|x|]_1^t = \lim_{t\to\infty}(\underset{\downarrow \infty}{\ln|t|} - \underset{\downarrow 0}{\cancel{\ln|1|}})$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$ is also divergent because the integral is divergent

* To figure out if positive/negative, take the derivative *

$\boxed{10.4}$ Comparison Tests (good for roots of polynomials in n)

Direct Comparison Test

* b_n could be $\frac{1}{n^J}$ where $J =$ (highest n power denom − highest n power num)

Suppose $0 \le a_n \le b_n$ eventually

- If Σb_n is convergent, then Σa_n is convergent
- If Σa_n is divergent, then Σb_n is divergent

Limit Comparison Test

Suppose Σa_n and Σb_n are series of positive terms, let $L = \lim_{n\to\infty} \frac{a_n}{b_n}$

- If L is finite and $\neq 0$, both converge or diverge
- If $L = 0$ and Σb_n is convergent, then Σa_n is convergent
- If $L = \infty$ and Σb_n is divergent, then Σa_n is divergent

Calculus 3 Midterm Review Continued

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$\boxed{10.5}$ Ratio Test and Root Test

- Positive Term Series

Ratio Test (useful for factorials & powers)
(don't use for n polynomials and ln)

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

- If $L<1$, then $\sum a_n$ is convergent
- If $L>1$, then $\sum a_n$ is divergent
- If $L=1$, then test is inconclusive

Root Test (don't use for rational or ln)

$$L = \lim_{n\to\infty} \sqrt[n]{a_n}$$

$\boxed{10.6}$ Alternating Series; Absolute & Conditional Convergence

Alternating Harmonic Series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges

Alternating Series Test

Suppose $\sum_{n=1}^{\infty} (-1)^{n+1} U_n$ is an alternating series

The series will converge if

- The sequence $\{U_n\}$ is eventually decreasing ($U_{n+1} \le U_n$)

and...

- $\lim_{n\to\infty} U_n = 0$

Theorem (Aabsolute Convergence ⇒ Convergence) [Test]

- If $\sum |a_n|$ is convergent then $\sum a_n$ is convergent

10.7 Power Series (has x)

A power series centered at a is an infinite series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n$

- Ratio test = $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$
- Root test = $\lim_{n\to\infty} \left|\sqrt[n]{a_n}\right|$
- Then find what values of x is $L<1$, which gives you interval of convergence
- Test endpoints by pluging into a_n and using other tests to find convergence
- Radius of convergence is distance from endpoints to center

* Always converges at the center *

* Function is only defined on the interval *

<u>Theorem</u> - On the interval, if you differentiate or integrate piece by piece, then the radius does not change.

10.8 Taylor Series

- First take derivatives to a degree
- Then plug center into derivatives
- Then use formula $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$
- Memorize common Taylor Series

10.9 Substitution

- If you see a modifier for x, substitute in the function

10.10 Binomial Series

- Taylor series for $f(x) = (1+x)^m$ | $\sum_{n=0}^{\infty} \binom{m}{n} x^n$ | $\binom{m}{n} = \frac{m(m-1)(m-2)\dots(m-n+1)}{n!}$ $\binom{m}{0} = 1$
- Radius of convergence is always 1 for binomial series

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Calculus 3 Midterm 2 Review

Parametric / Cycloid Equations

Area under one arch of cycloid $= \int_0^{2\pi a} y \, dx = \int_0^{2\pi} (a - a\cos t)(a - a\cos t) \, dt$

Length of one arch of cycloid $= \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \int_0^{2\pi} \sqrt{(a - a\cos t)^2 + (a\sin t)^2} \, dt$

standard cycloid $\Rightarrow$ $x = at - a\sin t$
$y = a - a\cos t$

Area of $\mathbb{R}^3$ Triangle

$P, Q, R \Rightarrow \quad \vec{PQ} = (Q - P)$ and $RR = (R - P)$

$\text{Area} = \frac{1}{2} bh$, so basically because cross product is a vector that conveys area created by the parallelogram of two vectors.

$\Rightarrow \quad \text{Area} = \frac{1}{2}(\vec{PQ} \times \vec{PR})$

Parametric Tangent

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}$$

Area of / Between Polar Curves

$A = \int_\alpha^\beta \frac{1}{2} (f(\theta))^2 \, d\theta$

* use r vs θ graph to draw *

$A = \int_\alpha^\beta \left(\frac{1}{2}(f(\theta))^2 - \frac{1}{2}(g(\theta))^2\right) d\theta$

amplitude

$y = a + b\cos b\theta$

move up or down — frequency

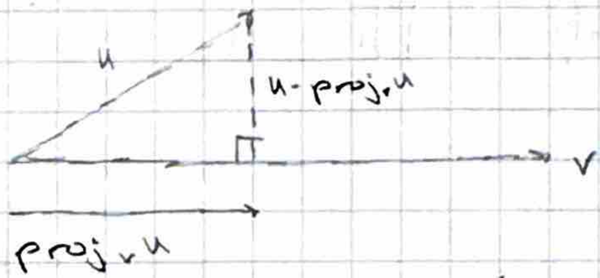
or...

$A = \frac{1}{2} \int_\alpha^\beta \left((f(\theta))^2 - (g(\theta))^2\right) d\theta$

Vector Projections

Scalar projection $\vec{u}$ on $\vec{v}$ = $\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|}$

Vector projection $\vec{u}$ on $\vec{v}$ = $\text{proj}_{\vec{v}}\vec{u}$ = $\left(\frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\right)\vec{v}$ because $\frac{\vec{v}}{\|\vec{v}\|^2}$ length (unit vector)



Show by calculation that $(u - \text{proj}_v u)\cdot \text{proj}_v u = 0$

$$\text{proj}_v u = \left(\frac{u\cdot v}{\|v\|^2}\right) \Rightarrow \left(u - \overbrace{\left(\frac{u\cdot v}{\|v\|^2}\right)}^{c} v\right)\cdot\left(\frac{u\cdot v}{\|v\|^2}\right)v \Rightarrow$$

$$(u - cv)\cdot cv = u\cdot(cv) - (cv)\cdot(cv) = c(u\cdot v) - c^2(v\cdot v) \Rightarrow$$

$$\frac{u\cdot v}{\|v\|^2}(u\cdot v) - \frac{(u\cdot v)^2}{(\|v\|^2)^2}\overbrace{(v\cdot v)}^{\|v\|^2} = \frac{(u\cdot v)^2}{\|v\|^2} - \frac{(u\cdot v)^2}{\|v\|^4}\|v\|^2 = \frac{(u\cdot v)^2}{\|v\|^2} - \frac{(u\cdot v)^2}{\|v\|^2} = 0 \checkmark$$

Dot Product (scalar)

$\vec{u}\cdot\vec{v} = \vec{u}_1\vec{v}_1 + \vec{u}_2\vec{v}_2 + \vec{u}_3\vec{v}_3 = \|\vec{u}\|\|\vec{v}\|\cos\theta$

$\vec{v}\cdot\vec{v} = \|v\|^2$ $\quad (\vec{u}\cdot\vec{v})\cdot\vec{w} = \vec{u}\cdot(\vec{v}\cdot\vec{w})$

$\hat{i}\cdot\hat{j},\ \hat{i}\cdot\hat{k},\ \hat{j}\cdot\hat{k} = 0$ (perpendicular)

$\hat{i}\cdot\hat{i},\ \hat{j}\cdot\hat{j},\ \hat{k}\cdot\hat{k} = 1$ (parallel / same)

Cross Product (vector) (right hand rule)

$$\vec{u}\times\vec{v} = \|\vec{u}\|\|\vec{v}\|\sin\theta = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = (u_2v_3 - u_3v_2)\hat{i} + \underbrace{(u_3v_1 - u_1v_3)}_{\text{or factor } (-1)}\hat{j} + (u_1v_2 - u_2v_1)\hat{k}$$

$\vec{v}\times\vec{v} = 0$ (parallel) $\checkmark$ $\quad (\vec{u}\times\vec{v})\times\vec{w} \neq \vec{u}\times(\vec{v}\times\vec{w})$

$\hat{i}\times\hat{j} = \hat{k}$ $\quad \hat{j}\times\hat{k} = \hat{i}$ $\quad \hat{k}\times\hat{i} = \hat{j}$ $\quad \hat{i}\to\hat{j}\to\hat{k}$

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Calculus 3 Final Review

Parametric Equations for L

$x = x_0 + t v_1$ $\quad\quad \vec{v} = \langle v_1, v_2, v_3 \rangle$

$y = y_0 + t v_2$ $\quad\quad$ *symmetry => equate coordinates*

$z = z_0 + t v_3$

Equation for a plane

$N = \langle n_1, n_2, n_3 \rangle$ = normal vector

$n_1(x - x_0) + n_2(y - y_0) + n_3(z - z_0) = 0$

Distance from a Point to a line

S = point in space = (s_1, s_2, s_3)

P = point on the line = (p_1, p_2, p_3)

$D = \dfrac{\|\vec{PS} \times \vec{v}\|}{\|\vec{v}\|}$ $\quad\quad \vec{v}$ = vector of line = $\langle v_1, v_2, v_3 \rangle$

Distance from a Point to a Plane

P = point of Normal $\quad\quad N$ = Normal

D is the absolute value of the scalar projection of $\vec{PS}$ on $\vec{N}$

$D = \left| \dfrac{\vec{PS} \cdot \vec{N}}{\|\vec{N}\|} \right|$

Curves in $\mathbb{R}^3$ space

$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$ $\quad\quad \vec{r}(t)$ is smooth if $x'(t)$, $y'(t)$, and $z'(t)$ are all continuous

$\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$ $\quad\quad \|\vec{r}(t)\|$ = speed

Arc Length / Function

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt = \int_a^b \|\vec{v}(t)\|\, dt$$

$$S(t) = \int_a^t \|\vec{v}(\tau)\|\, d\tau$$

Curvature

Unit Tangent vector $= \vec{T}(t) = \frac{r'(t)}{\|r'(t)\|} = \frac{v(t)}{\|v(t)\|}$

curvature $= k(t) = \left\|\frac{d\vec{T}}{ds}\right\| = \frac{\|\vec{T}'(t)\|}{\|\vec{r}'(t)\|} = \frac{\|\vec{r}'(t) \times \vec{r}''(t)\|}{\|\vec{r}'(t)\|^3} = \frac{\|\vec{v}(t) \times \vec{a}(t)\|}{\|\vec{v}(t)\|^3}$

curvature in the x-y plane $= k(x) = \frac{|f''(x)|}{(1+(f'(x))^2)^{\frac{3}{2}}}$

Extra

$\theta = \cos^{-1}\left(\frac{v \cdot u}{|v||u|}\right)$ $\qquad$ $\theta = \sin^{-1}\left(\frac{v \times u}{|v||u|}\right)$

Area of parallelogram $= |u \times v| =$ abs $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

Volume of parallelepiped $= |u \times v| \cdot w = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

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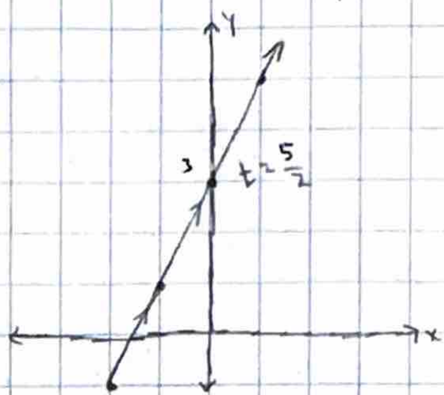
-1 section #

19/20

11.1 # 3,5,13,19,21,28

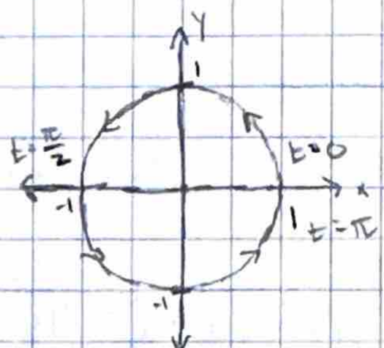
3. $x = 2t - 5$, $y = 4t - 7$, $-\infty < t < \infty$

$t = \frac{1}{2}x + 2\frac{1}{2} \Rightarrow y = 4(\frac{1}{2}x + 2\frac{1}{2}) - 7 \Rightarrow y = 2x + 10 - 7 \Rightarrow y = 2x + 3$



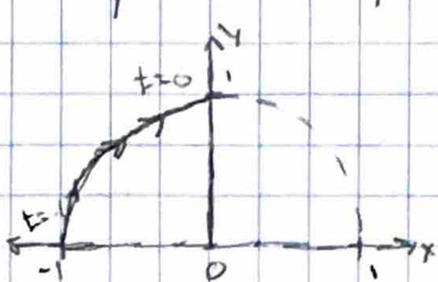
5. $x = \cos 2t$, $y = \sin 2t$, $0 \le t \le \pi$

$\cos^2 2t + \sin^2 2t = x^2 + y^2 = 1$



13. $x = t$, $y = \sqrt{1 - t^2}$, $-1 < t < 1$

$\Rightarrow y = \sqrt{1 - x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 + y^2 = 1$



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19. $x^2 + y^2 = a^2$

$\Rightarrow$ a) $x = a\cos t$, $y = -a\sin t$, $0 \leq t \leq 2\pi$

b) $x = a\cos t$, $y = a\sin t$, $0 \leq t \leq 2\pi$

c) $x = a\cos t$, $y = -a\sin t$, $0 \leq t \leq 4\pi$

d) $x = a\cos t$, $y = a\sin t$, $0 \leq t \leq 4\pi$

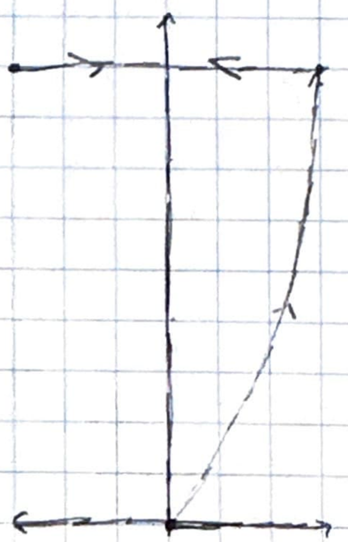
21. $(-1,-3)$, $(4,1)$ $\Rightarrow$ slope $= \frac{1+3}{4+1} = \frac{4}{5}$

$x = -1 + at$, $y = -3 + bt$, line goes through $(-1,-3)$ at $t = 0$

for line to hit $(4,1)$ when $t = 1$ $\Rightarrow$ $4 = -1 + a$, $1 = -3 + b$

$\Rightarrow a = 5$, $b = 4$ $\Rightarrow$ $\boxed{x = -1 + 5t,\ y = -3 + 4t,\ 0 \leq t \leq 1}$

28. $y = x^2$, $(0,0)$, $(3,9)$, $(3,9) \Leftrightarrow (-3,9)$



x stays between $3, -3$

$\Rightarrow x = 3\sin t \Rightarrow y = (3\sin t)^2 = 9\sin^2 t$

$\Rightarrow \boxed{x = 3\sin t,\ y = 9\sin^2 t,\ 0 \leq t\ \infty}$

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11.2 # 3, 14, 21, 41, 47a

* Show that the tangent line to the cycloid at point P passes through the top of the rolling circle*

3. $x = 4\sin t$, $y = 2\cos t$, $t = \frac{\pi}{4}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Rightarrow \frac{dy}{dt} = -2\sin t$, $\frac{dx}{dt} = 4\cos t$

$\Rightarrow \frac{dy}{dx} = \frac{-2\sin t}{4\cos t} = -\frac{1}{2}\tan t$, $t = \frac{\pi}{4} \Rightarrow \frac{dy}{dx} = -\frac{1}{2}(1) = -\frac{1}{2}$

$t = \frac{\pi}{4} \Rightarrow x = 4\sin(\frac{\pi}{4}) = 2\sqrt{2}$, $y = 2\cos(\frac{\pi}{4}) = \sqrt{2} \Rightarrow (2\sqrt{2}, \sqrt{2})$

$\Rightarrow$ tangent line is $y - \sqrt{2} = -\frac{1}{2}(x - 2\sqrt{2}) = \boxed{y = -\frac{1}{2}x + 2\sqrt{2}}$

$\frac{dy'}{dt} = -\frac{1}{2}\sec^2 t \Rightarrow \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{-\frac{1}{2}\sec^2 t}{4\cos t} = -\frac{1}{8\cos^3 t}$

$\Rightarrow \frac{d^2y}{dx^2}(t = \frac{\pi}{4}) = \boxed{-\frac{\sqrt{2}}{4}}$

14. $x = t + e^t$, $y = 1 - e^t$, $t = 0$

for $t = 0$, $x = 1$, $y = 0$

$\frac{dx}{dt} = 1 + e^t$, $\frac{dy}{dt} = -e^t \Rightarrow \frac{dy}{dx} = \frac{-e^t}{1+e^t}$, at $t = 0$, $\frac{dy}{dx} = \frac{-1}{2}$

$\Rightarrow \boxed{y = -\frac{1}{2}(x-1)}$ = tangent line

$\frac{dy'}{dt} = \frac{-e^t}{(1+e^t)^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{-e^t}{(1+e^t)^3} \Rightarrow$ at $t = 0$ $\frac{d^2y}{dx^2} = \boxed{-\frac{1}{8}}$

21. cycloid: $x = a(t - \sin t)$ $y = a(1 - \cos t)$

- area under one arch of the cycloid is 3 times the area of the rolling circle

$A = \int_0^{2\pi} y\,dx = \int_0^{2\pi} a(1-\cos t)a(1-\cos t)dt = a^2\int_0^{2\pi}(1-\cos t)^2\,dt = a^2\int_0^{2\pi}(1 - 2\cos t + \cos^2 t)dt$

$= a^2\int_0^{2\pi}(1 - 2\cos t + \frac{1+\cos 2t}{2})dt = a^2\int_0^{2\pi}(\frac{3}{2} - 2\cos t + \frac{1}{2}\cos 2t)dt = a^2\left[\frac{3}{2}t - 2\sin t + \frac{1}{4}\sin 2t\right]_0^{2\pi}$

$= a^2(3\pi - 0 + 0) - 0 = \boxed{3\pi a^2}$ ✓

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41. a) $\frac{dx}{dt} = -2\sin 2t$, $\frac{dy}{dt} = 2\cos 2t \Rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2} = 2 \rightarrow L = \int_0^{\frac{\pi}{2}} 2\,dt = \left[2t\right]_0^{\frac{\pi}{2}} = \boxed{\pi}$ ✓

b) $\frac{dx}{dt} = \pi\cos\pi t$, $\frac{dy}{dt} = -\pi\sin\pi t \rightarrow \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{(\pi\cos\pi t)^2 + (-\pi\sin\pi t)^2} = \pi \Rightarrow \text{length} = \int_{-1/2}^{1/2} \pi\,dt = \left[\pi t\right]_{-1/2}^{1/2} = \boxed{\pi}$ ✓

47. a) $x = a(t - \sin t)$, $y = a(1 - \cos t)$, $0 \le t \le 2\pi$

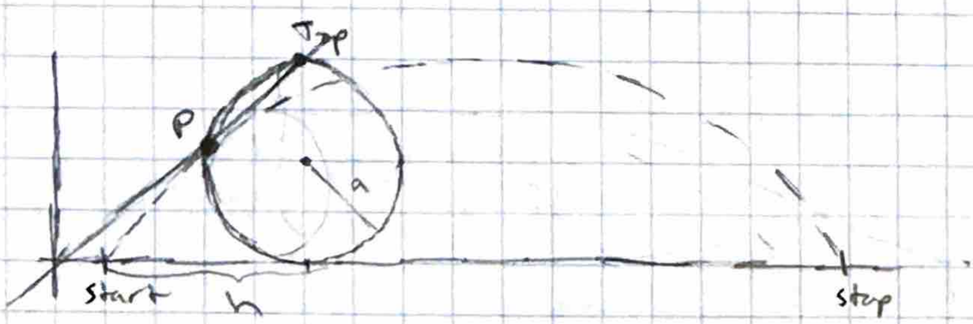
$\frac{dx}{dt} = a(1-\cos t)$, $\frac{dy}{dt} = a\sin t \Rightarrow L = \int_0^{2\pi} \sqrt{(a(1-\cos t))^2 + (a\sin t)^2}\,dt$

$= \int_0^{2\pi} \sqrt{a^2 - 2a^2 + a^2\cos^2 t + a^2\sin^2 t}\,dt = a\sqrt{2}\int_0^{2\pi}\sqrt{1-\cos t}\,dt =$

$a\sqrt{2}\int_0^{2\pi}\sqrt{2\sin^2\left(\frac{1}{2}\right)}\,dt = 2a\int_0^{2\pi}\sin\left(\frac{t}{2}\right)dt = \left[-4a\cos\left(\frac{t}{2}\right)\right]_0^{2\pi}$

$= -4a\cos\pi + 4a\cos(0) = \boxed{8a}$ ✓

Tangent line at point p passes through the top of the rolling circle.



$\text{Top} = (h, 2a) \Rightarrow \left(\frac{3\pi a}{4}, 2a\right)$

$2a = (\sqrt{2}-1)\left(\frac{3\pi a}{4}\right) - \frac{3\pi a}{2\sqrt{2}} + 2a + \frac{3\pi a}{4}$

$2a = \cancel{\frac{3\pi a}{2\sqrt{2}}} - \cancel{\frac{3\pi a}{4}} - \cancel{\frac{3\pi a}{2\sqrt{2}}} + 2a + \cancel{\frac{3\pi a}{4}}$

$2a = 2a$ ✓ $\Rightarrow$ tangent line passes through the top of the cycloid

19. $r = \sin 2\theta$; $\theta = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}$

$\theta = \frac{\pi}{4} \Rightarrow r = 1 \Rightarrow (1, \frac{\pi}{4})$
$\theta = -\frac{\pi}{4} \Rightarrow r = -1 \Rightarrow (-1, -\frac{\pi}{4})$
$\theta = \frac{3\pi}{4} \Rightarrow r = -1 \Rightarrow (-1, \frac{3\pi}{4})$
$\theta = -\frac{3\pi}{4} \Rightarrow r = 1 \Rightarrow (1, -\frac{3\pi}{4})$

$$\text{slope} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{2\cos 2\theta \sin\theta + r\cos\theta}{2\cos 2\theta \cos\theta - r\sin\theta}$$

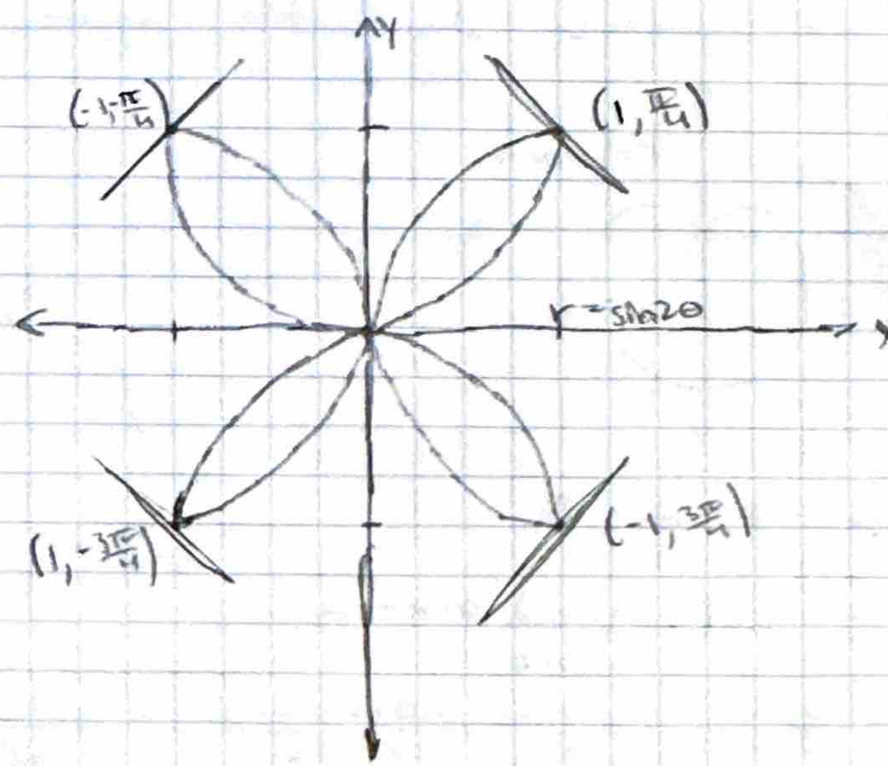
$r' = \frac{dr}{d\theta} = 2\cos 2\theta$

slope $(1, \frac{\pi}{4}) = -1$

slope $(-1, -\frac{\pi}{4}) = 1$

slope $(-1, \frac{3\pi}{4}) = 1$

slope $(1, -\frac{3\pi}{4}) = -1$



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11.5 # 6, 9, 11, 15

6. $r = \cos 3\theta$

$$A = \int_{-\pi/6}^{\pi/6} \frac{1}{2}(\cos 3\theta)^2 \, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \cos^2 3\theta \, d\theta = \frac{1}{2}\int_{-\pi/6}^{\pi/6} \frac{1+\cos 6\theta}{2} \, d\theta$$

$$= \frac{1}{4}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1+\cos 6\theta)\, d\theta = \frac{1}{4}\left[\theta + \frac{1}{6}\sin 6\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

$$= \frac{1}{4}\left(\frac{\pi}{6}+0\right) - \frac{1}{4}\left(-\frac{\pi}{6}+0\right) = \boxed{\frac{\pi}{12}}$$

9. $r = 2\cos\theta \qquad r = 2\sin\theta$

$2\cos\theta = 2\sin\theta \Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$

$$\Rightarrow A = 2\int_0^{\frac{\pi}{4}} \frac{1}{2}(2\sin\theta)^2 \, d\theta = \int_0^{\frac{\pi}{4}} 4\sin^2\theta \, d\theta$$

$$= \int_0^{\pi/4} 4\left(\frac{1-\cos 2\theta}{2}\right) d\theta = \int_0^{\frac{\pi}{4}} (2 - 2\cos 2\theta)\, d\theta$$

$$= \left[2\theta - \sin 2\theta\right]_0^{\frac{\pi}{4}} = \boxed{\frac{\pi}{2} - 1}$$

11. $r = 2(1+\cos\theta) \qquad r = 2(1-\cos\theta)$

$\Rightarrow 2 = 2(1-\cos\theta)$

$\Rightarrow \cos\theta = 0 \Rightarrow \pm\frac{\pi}{2}$

$$\Rightarrow A = 2\int_0^{\pi/2} \frac{1}{2}\left[2(1-\cos\theta)\right]^2 d\theta + \frac{1}{2} \text{ area of circle}$$

$$= \int_0^{\frac{\pi}{2}} 4(1 - 2\cos\theta + \cos^2\theta)\, d\theta + \left(\frac{1}{2}\pi\right)(2)^2$$

$$= \int_0^{\frac{\pi}{2}} 4\left(1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right) d\theta + 2\pi$$

$$= \int_0^{\pi/2} (4 - 8\cos\theta + 2 + 2\cos 2\theta)\, d\theta + 2\pi$$

$$= \left[6\theta - 8\sin\theta + \sin 2\theta\right]_0^{\pi/2} + 2\pi = \boxed{5\pi - 8}$$

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12.1 # 1, 5, 9, 12, 15, 17, 21, 24, 41, 51, 53, 55

1. $x=2,\ y=3$

$\Rightarrow$ Line through the point $(2,3,0)$ parallel to z-axis

5. $x^2+y^2=4,\ z=0$

$\Rightarrow$ circle with radius $r=2$ in the x-y plane

9. $x^2+y^2+z^2=1,\ x=0$

$\Rightarrow$ circle with radius $r=1$ in the y-z plane

12. $x^2+(y-1)^2+z^2=4,\ y=0$

$\Rightarrow$ circle with radius $r=\sqrt{3}$ in the x-z plane

15. $y=x^2,\ z=0$

$\Rightarrow$ The parabola $y=x^2$ in the x-y plane

17. a) $x\geq 0,\ y\geq 0,\ z=0$

$\Rightarrow$ The first quadrant of the x-y plane

b) $x\geq 0,\ y\leq 0,\ z=0$

$\Rightarrow$ The fourth quadrant of the x-y plane

21. a) $1\leq x^2+y^2+z^2\leq 4$

$\Rightarrow$ The solid enclosed between the sphere of radius 1 and radius 2 centered at the origin

b) $x^2+y^2+z^2\leq 1,\ z\geq 0$ $\Rightarrow$ The solid upper hemisphere of radius 1 centered at the origin

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10.9 # 1, 3, 5, 7, 8, 9, 13, 16, 19, 20, 23, 29, 31

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1. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\Rightarrow e^{-5x} = 1 + (-5x) + \frac{(-5x)^2}{2!} + \frac{(-5x)^3}{3!} \dots = 1 - 5x + \frac{5^2x^2}{2!} - \frac{5^3x^3}{3!} \dots$

$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^n 5^n}{n!}}$

3. $\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}(-1)^n}{(2n+1)!}$

$\Rightarrow -5\sin(x) = 5\left[(-x) - \frac{(-x)^3}{3!} + \frac{(-x)^5}{5!} - \frac{(-x)^7}{7!} + \frac{(-x)^9}{9!} \dots\right]$

$= 5\left[-x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \frac{x^9}{9!} \dots\right]$

$= 5\sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)}}{(2n+1)!} = \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{(2n+1)} 5}{(2n+1)!}}$

5. $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

$\Rightarrow \cos 5x^2 = 1 - \frac{(5x^2)^{2n}}{2!} + \frac{(5x^2)^{4n}}{4!} - \frac{(5x^2)^{6n}}{6!} \dots$

$= 1 - \frac{5^{2n}x^{4n}}{2!} + \frac{5^{4n}x^{6n}}{4!} - \frac{5^{6n}x^{8n}}{6!} \dots$

$= \boxed{\sum_{n=0}^{\infty} \frac{5^{2n} x^{4n} (-1)^n}{n!}}$

7. $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$

$\Rightarrow \ln(1+x^2) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x^2)^n}{n} = \boxed{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}}$

8. $\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

$\Rightarrow \tan^{-1}(3x^4) = \sum_{n=0}^{\infty} (-1)^n \frac{(3x^4)^{2n+1}}{2n+1} = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{3^{2n+1} x^{8n+4}}{2n+1}}$

9. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$\Rightarrow \frac{1}{1+\frac{3}{4}x^3} = \sum_{n=0}^{\infty} \left(-\frac{3}{4}x^3\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{3^n x^{3n}}{4^n} (-1)^n}$

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10.10 # 2, 4, 5, 7, 10

2. $f(x) = (1+x)^{\frac{1}{3}}$

$$\Rightarrow \sum_{n=0}^{\infty} \binom{\frac13}{n} x^n = \binom{\frac13}{0} + \binom{\frac13}{1} x + \binom{\frac13}{2} x^2 + \binom{\frac13}{3} x^3$$

$$= \boxed{1 + \frac{1}{3}x + \frac{(\frac13)(-\frac23)}{2} x^2 + \frac{(\frac13)(-\frac23)(-\frac53)}{6} x^3}$$

4. $f(x) = (1-2x)^{\frac{1}{2}}$

$$\Rightarrow \sum_{n=0}^{\infty} \binom{\frac12}{n} 2^n x^n = \binom{\frac12}{0} + \binom{\frac12}{1} 2x + \binom{\frac12}{2} 4x^2 + \binom{\frac12}{3} 8x^3$$

$$= 1 + x + \frac{(\frac12)(\frac12 - 1)}{2} 4x^2 + \frac{(\frac12)(\frac12-1)(\frac12-2)}{6} 8x^3$$

$$= \boxed{1 - x - \frac{1}{2}x^2 - \frac{1}{2}x^3}$$

5. $f(x) = (1+\frac{x}{2})^{-2}$

$$\Rightarrow \sum_{n=0}^{\infty} \binom{-2}{n} \frac{x^n}{2^n} = \binom{-2}{0} + \binom{-2}{1}\frac{x}{2} + \binom{-2}{2}\frac{x^2}{4} + \binom{-2}{3}\frac{x^3}{8}$$

$$= 1 + \frac{-2}{1!}\frac{x}{2} + \frac{(-2)(-2-1)}{2!}\frac{x^2}{4} + \frac{(-2)(-2-1)(-2-2)}{3!}\frac{x^3}{8}$$

$$= \boxed{1 - x + \frac{3}{4}x^2 - \frac{1}{2}x^3}$$

7. $f(x) = (1+x^3)^{-1/2}$

$$\Rightarrow \sum_{n=0}^{\infty} \binom{-\frac12}{n} x^{3n} = \binom{-\frac12}{0} + \binom{-\frac12}{1} x^3 + \binom{-\frac12}{2} x^6 + \binom{-\frac12}{3} x^9$$

$$= 1 + \frac{-\frac12}{1} x^3 + \frac{(-\frac12)(-\frac12-1)}{2} x^6 + \frac{(-\frac12)(-\frac12-1)(-\frac12-2)}{6} x^9$$

$$= \boxed{1 - \frac{1}{2}x^3 + \frac{3}{8}x^6 - \frac{5}{16}x^9}$$

10. $f(x) = \frac{x}{\sqrt[3]{1+x}} = (1+x)^{-\frac13} x$

$$\Rightarrow x\sum_{n=0}^{\infty} \binom{-\frac13}{n} x^n = x\left(\binom{-\frac13}{0} + \binom{-\frac13}{1} x + \binom{-\frac13}{2} x^2 + \binom{-\frac13}{3} x^3\right)$$

$$= x\left(1 + -\frac{1}{3}x + \frac{(-\frac13)(-\frac43)}{2} x^2 + \frac{(-\frac13)(-\frac43)(-\frac73)}{6} x^3\right)$$

$$= \boxed{x - \frac{1}{3}x^2 + \frac{2}{9}x^3 - \frac{14}{81}x^4}$$

Section 54 Name Elassaad, Mohammad Points "19" 2

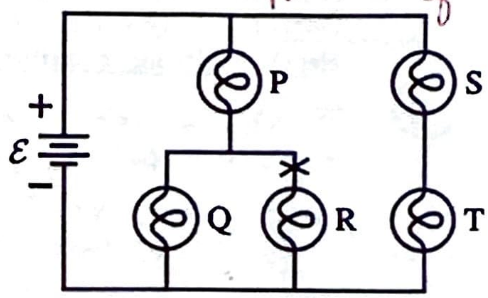
(last, first)

noted in gradebook,

Physics 133 – WA 8 – Spring 2019

1. **[10 pts total]**

   A. The circuit shown has an ideal battery and five identical **non-ohmic** bulbs P, Q, R, S, and T. (The "X" above bulb R is for parts b, c, d.)



   a. [4 pts] Rank all the bulbs from brightest to dimmest using the symbols ">" and/or "=". **Using physics, briefly explain.**

   P > S = T > Q = R

   4 P is the brightest because it gets the most voltage therefore the most current. The current there then splits at the junction making Q and R the weakest. Q = R because they are in parallel and identical. S and T are not as bright as P but brighter than Q or R because S and T are in series so they split voltage evenly, about ½ each, while Q or R get less than 1/2 the voltage each.

For the next three parts, the wire above bulb R is cut with scissors at the "X," creating a gap there.

   b. [2 pts] Does the brightness of bulb S *increase*, *decrease*, or *stay the same* (circle one)? **Explain.**

   2 S stays the same because S and the loop where the cut is are in parallel. Therefore the brightness of S is independent of what goes on at the cut.

   c. [2 pts] Does the brightness of bulb P *increase*, *decrease*, or *stay the same* (circle one)? **Explain.**

   2 P decreases because now it is in series with only one bulb and there is no junction right after it. This means the voltage goes from above ½ the battery to exactly ½, P will = Q.

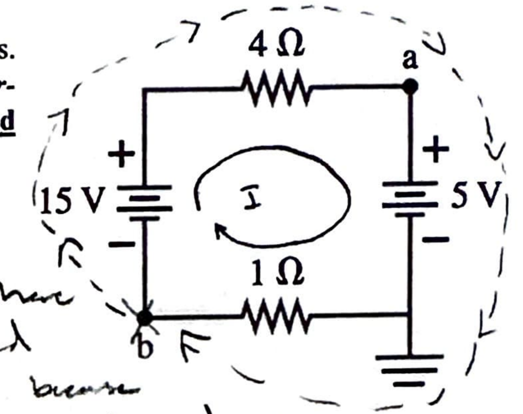
   d. [2 pts] Does the brightness of bulb Q *increase*, *decrease*, or *stay the same* (circle one)? **Explain.**

   2 Q increases because originally it was in parallel with R, but now in series with P. This means Q and P now have to evenly add up to battery voltage which makes them equal and both ½ battery voltage. Before the cut Q had less than ½ battery voltage.

Section 54 Name Elassaad, Mohammad Points ______

(last, first)

2. [10 pts] The circuit shown consists of two ideal batteries and two resistors.



a. [2 pts] Determine the magnitude *and* direction (*clockwise* or *counter-clockwise*) of the current flowing in the circuit. **Show all work, and clearly justify any equation you use or calculation you make.**

First I set current to clockwise. Then I follow the path I drew and use V=IR and K's loop rule. There are no junctions so we don't need K's junction rule. We can use these because resistors are ohmic, which follow ohms law.

$\sum(\Delta V)_i = 0 = +15\,(\text{battery}) - 4I\,(\text{resistor}) - 5\,(\text{battery}) - 1I\,(\text{resistor})$

2

$\Rightarrow 0 = 15 - 4I - 5 - I = 10 - 5I \Rightarrow -10 = -5I \Rightarrow \boxed{I = 2 \text{ amps}}$

We know $\boxed{\text{clockwise}}$ is correct because our calculated current is positive.

b. [2 pts] What is the magnitude of the potential difference, $|\Delta V_{ab}|$, between locations "a" and "b"? **Show your work.**

2 $|\Delta V_{ab}| = |-5\,(\text{battery}) - 1I\,(\text{resistor})| = |-5 - 1(2)| = |-5-2| = |-7| = 7$

$\Rightarrow \boxed{|\Delta V_{ab}| = 7 \text{ volts}}$

c. [2 pts] Which location, "a" or "b," is at the higher potential? Or are they at the same potential? **Briefly explain how you know.**

1 "a" is at a higher potential than "b" because in direction of current to get to "b" from "a" you have to lose 5V from the battery then go through a resistor. To get to "a" from "b" you gain 15 V from a battery and go through a different resistor.

d. [2 pts] How much energy per second is transferred from the two batteries to the resistors? **(Show your work.)**

power = J/s = W

$P_R = I\Delta V_R = I^2R = \frac{(\Delta V_R)^2}{R}$ $R_{eq,s} = 4\Omega + 1\Omega = 5\Omega$

$I = 2$ amps

2 $P_R = I^2R = (2\text{ amps})^2\, 5\Omega = 20W$

$\Rightarrow \boxed{20 \text{ Joules per second}}$

e. [2 pts] Is there a non-zero net electric field inside of the 4 Ω resistor? If yes, which direction does it point? If no, why is it zero? **Briefly explain how you know.**

Yes, there has to be a non-zero net electric field, 2 otherwise the circuit would be at electrostatic equilibrium and there will be no current. The treasure people put more positive charges on the left of the resistor, and the current goes to the right. This means voltage is lost. We know electric fields always point from high to low potential. Therefore the direction is to the right.

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Calc 2

Math 1412

3/20/14

Calculus Chapters 5 + 6 Review

Fundamental Theorem of Calculus (Definite)

$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where } F = \text{antiderivative}$$

Indefinite Integral

$$\int f(x)\,dx = F(x) + C \quad \text{where } F = \text{antiderivative}$$

When Inner Variable is Multiplied by Constant

$$\int \cos(ax)\,dx = \frac{\sin(ax)}{a} + C \quad \text{(chain rule)}$$

Integrals of $\sin^2 x$ and $\cos^2 x$

Use identities:

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Area Between Curves

$\int (\text{top} - \text{bottom}) \Rightarrow$ break if necessary

Area Between Curves Continued

$$\text{Area} = \int_a^b \underbrace{f(x) - g(x)}_{\text{top - bottom}}\, dx$$

$$\text{Area} = \int_c^d \underbrace{f(y) - g(y)}_{\text{right - left}}\, dy$$

The Slice Method

* For volume, bounds only of repeated (consistent) shape *

$$V = \int_a^b A(x)\,dx = \int_a^b (\text{Area of Cross-Section})\,dx$$

for solids of revolution

$$V = \int_a^b \pi R^2\,dx = \int_a^b \pi\,(f(x))^2\,dx \quad \text{(about x-axis)}$$

$$V = \int_c^d \pi R^2\,dy = \int_a^b \pi\,(f(y))^2\,dy \quad \text{(about y-axis)}$$

The Washer Method

$$V = \int_a^b A(x)\,dx = \int_a^b \pi R^2 - \pi r^2\,dx \quad \text{vertical slice} \perp \text{x-axis}$$

$$V = \int_c^d A(y)\,dy = \int_c^d \pi R^2 - \pi r^2\,dy \quad \text{horizontal slice} \perp \text{y-axis}$$

Shell Method

- For Disk/Washer we sliced the solid with planes
- For shell we slice our solid using circular cylinders

$V = \int_a^b 2\pi r h\, dx = \int_a^b 2\pi(\text{shell radius})(\text{shell height})\, dx$ if (upright) vertical cylinder

$V = \int_c^d 2\pi r h\, dy$ if lying down cylinder (horizontal)

Arc length (length of the curve)

$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \qquad L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

Area of Surfaces of Revolution (Hollow object)

$S = \int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \qquad S = \int_c^d 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

(revolve around x-axis) (revolve around y-axis)

Work/Pumping Liquids from Containers

$W = Fd = \int_a^b F(x)\, dx \qquad \Delta V = \begin{pmatrix}\text{area of} \\ \text{cross section}\end{pmatrix} \Delta y$

$F(y) = (\Delta V)(\text{density})$

$W = F(y)(\text{distance to move slab}) = \int_c^d \begin{pmatrix}\text{area of} \\ \text{cross} \\ \text{section}\end{pmatrix}(\text{density})(\text{distance to move slab})$

Moments and Centers of Mass

Moment about the x-axis...

$M_x = \sum m_k y_k$ (signed distance to x-axis ⇒ measures up/down tilt)

Moment about the y-axis...

$M_y = \sum m_k x_k$ (signed distance to y-axis ⇒ measures right/left tilt)

Center of mass = $(\bar{x}, \bar{y})$...

$\bar{x} = \frac{\sum m_k x_k}{\sum m_k} = \frac{M_y}{M}$ $\quad \bar{y} = \frac{\sum m_k y_k}{\sum m_k} = \frac{M_x}{M}$

Center of Mass of a Thin Flat Plate (Vertical/Horizontal Strip Method)

$M = \int_a^b (\text{density})(\text{length of strip})\,dx = \int_a^b \delta(x) f(x)\,dx$

(average)

$M_x = \int_a^b (\text{density})\begin{pmatrix}\text{length of}\\ \text{strip}\end{pmatrix}\begin{pmatrix}\text{distance}\\ \text{to x-axis}\end{pmatrix} dx = \int_a^b \delta(x) f(x) \left(\frac{f(x)}{2}\right) dx$

$M_y = \int_a^b (\text{density})\begin{pmatrix}\text{length of}\\ \text{strip}\end{pmatrix}\begin{pmatrix}\text{distance}\\ \text{to y-axis}\end{pmatrix} dx = \int_a^b \delta(x) f(x)(x)\,dx$

$CM = (\bar{x}, \bar{y}) \quad \bar{x} = \frac{M_y}{M}, \quad \bar{y} = \frac{M_x}{M}$

Flip bounds + integral + variables for horizontal

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3/3/19

Calculus Chapter 7 Review

$\boxed{7.1}$ Inverse Functions

One to One - Horizontal Line Test
Inverse - Reflection Across $y=x$ - switch x and y (solve)

Derivative of Inverse

$$(f^{-1})'(\text{output}) = \frac{1}{f'(\text{input})} \iff (f^{-1})'(b) = \frac{1}{f'(a)}$$

(look at a point) (find corresponding first)

$\boxed{7.2}$ Natural Logarithms

$\ln(x) = \int_1^x \frac{1}{t}\,dt$ $\quad \ln(0) = -\infty \quad \ln(1) = 0$

Derivative of $y = \ln x$

$$\frac{d}{dx}(\ln(x)) = \frac{d}{dx}\int_1^x \frac{1}{t}\,dt \overset{\text{FTC Part I}}{=} \frac{1}{x}$$

$$\frac{d}{dx}(\ln(f(x))) = \frac{1}{f(x)} \cdot f'(x)$$

Properties of $y = \ln(x)$ $\Rightarrow$
1. $\ln(bx) = \ln(b) + \ln(x)$
2. $\ln\left(\frac{b}{x}\right) = \ln(b) - \ln(x)$
3. $\ln\left(\frac{1}{x}\right) = -\ln(x)$
4. $\ln(x^r) = r\ln(x)$, r is rational

Calculus Chapter 7 Review Continued

New Integrals

1. $\int \frac{1}{x} dx = \ln|x| + C$

2. $\int \tan x \, dx = \ln|\sec x| + C$

3. $\int \cot x \, dx = \ln|\sin x| + C$

4. $\int \sec x \, dx = \ln|\sec x + \tan x| + C$

5. $\int \csc x \, dx = -\ln|\csc x + \cot x| + C$

Logarithmic Differentiation

Step 1 - Take ln of both sides

Step 2 - Use properties of ln

Step 3 - Differentiate

Step 4 - Solve for $\frac{dy}{dx}$

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Calculus Chapter 7 Review Continued

$\boxed{7.3}$ Exponential Functions

$y = e^x$ is the inverse of $y = \ln x$

$e^{\ln(x)} = x$ $\qquad$ $\ln(e^x) = x$

Properties of $y = e^x$ $\Rightarrow$ 1. $e^{x_1} e^{x_2} = e^{x_1 + x_2}$

2. $e^{-x} = \frac{1}{e^x}$
3. $\frac{e^{x_1}}{e^{x_2}} = e^{x_1 - (x_2)}$
4. $(e^x)^r = e^{xr}$, r is rational

Derivative of $y = e^x$

$\frac{d}{dx}(e^x) = e^x$ $\qquad$ $\frac{d}{dx}(e^{f(x)}) = e^{f(x)} \cdot f'(x)$

Integral of $y = e^x$

$\int e^x \, dx = e^x + C$

General Exponential Functions

If $a > 0$, $a^x = e^{x \ln a}$

Calculus Chapter 7 Review Continued

Derivative of $y = a^x$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(a^{f(x)}) = a^{f(x)} \cdot \ln a \cdot f'(x)$$

Integrals of $y = a^x$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

If there is a variable in the exponent and base, then use logarithmic differentiation

Logarithms with Base a

$$\log_a x = \frac{\ln x}{\ln a}$$

Derivative of $y = \log_a x$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\log_a f(x)) = \frac{1}{f(x) \ln a} \cdot f'(x)$$

$$y' = \frac{dy}{dx} = \left(\frac{1}{f(x)}\right)\left(\frac{1}{\ln a}\right)\left(f'(x)\right)$$

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Calculus Chapter 7 Review Continued

$\boxed{7.4}$ Exponential Change

$y(t) = y_0 e^{-kt}$ (decay) half life $= t = \frac{\ln 2}{k}$

$y(t) = y_0 e^{kt}$ (growth)

$\boxed{7.5}$ L'Hopital's Rule

Check limit of top and bottom, if $\frac{0}{0}$ or $\frac{\infty}{\infty}$ differentiate top and bottom to find limit

You can pull the limit inside a system

$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin x}} = \lim_{x\to 0^+} \sqrt{\frac{x}{\sin x}} = \sqrt{\lim_{x\to 0^+} \frac{x}{\sin x}}$ | Same with ln

Simplify as much as possible between LR

Convert to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ if not already

I. $\frac{0}{0}, \frac{\infty}{\infty}$ L'Hopital's Rule $\lim \frac{f'}{g'} = \lim \frac{f}{g}$

II. $0 \cdot \infty, \infty - \infty$ Convert to $\frac{0}{0}$ or $\frac{\infty}{\infty}$ $\left(f_g^{0\cdot\infty} = \frac{f}{(1/g)} = \frac{g}{(1/f)} \right)$

III. $1^\infty, 0^0, \infty^0$ Take ln to convert to $0 \cdot \infty$, compute using II, DONT forget to Exponentiate

Calculus Chapter 7 Review Continued

$\boxed{7.6}$ Inverse Trig Functions

Derivative Rule for Inverses

$$(f^{-1})'(b) = \frac{1}{f'(a)} \qquad \text{where } f(a)=b$$

b: output a: input

Derivatives

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad |u|<1 \qquad y = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\frac{d}{dx}\cos^{-1}u = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \quad |u|<1 \qquad y = [0, \pi]$$

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx} \qquad y = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\frac{d}{dx}\cot^{-1}u = -\frac{1}{1+u^2}\frac{du}{dx} \qquad y = [0, \pi]$$

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} \quad |u|>1 \qquad y = [0, \pi]$$

$$\frac{d}{dx}\csc^{-1}u = -\frac{1}{|u|\sqrt{u^2-1}} \quad |u|>1 \qquad y = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Integrals $(a \neq 0)$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad u^2 < a^2$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C \qquad \text{all } u$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \qquad |u|>a>0$$

Mohammad Blessoad
Calc 2
Math 1112
3/21/19

Calculus Chapter 8 Review

Integration by Parts

- If you see an integral with two functions multiplied

$\int u\,dv = uv - \int v\,du$ $\quad u = f(x) \quad dv = g'(x)$
$\quad du = f'(x) \quad v = g(x)$

L logarithmic
I Inverse trig
P Polynomial
E Exponential
T trig

Order to choose u

* u should get simpler when you take the derivative *

Trig Integrals

$\sin^2 x = \frac{1-\cos 2x}{2}$ and $\cos^2 x = \frac{1+\cos 2x}{2}$

or substitute

$\int \sin^m x \cos^n x\,dx$ pull out something to $\sin^2 x + \cos^2 x = 1$
then integrate with u-sub

- If square root or other form use trig substitution

$\int \tan^m x \sec^n x\,dx$ pull out something to $1 + \tan^2 x = \sec^2 x$

Trig substitutions (almost looks like inverse trig)

- put x in terms of θ to eliminate square root with sub
- integrate with other subs and put back in terms of x (use $\triangle$)

Partial Fractions (Integrate $\frac{\text{polynomial}}{\text{polynomial}}$)

Given $\int \frac{f(x)}{g(x)} dx \quad deg(f) < deg(g)$

Step 1 - Factor $g(x)$ into linear / irreducible quadratic terms

Step 2 - write $\frac{f(x)}{g(x)}$ as a sum of partial fractions where:

(I) Each linear term $(x-r)^m$ contributes

$$\frac{A_1}{(x-r)} + \frac{A_2}{(x-r)^2} + \frac{A_3}{(x-r)^3} + \dots + \frac{A_m}{(x-r)^m}$$

(II) Each irreducible quadratic term $(x^2+px+q)^n$ contributes

$$\frac{B_1x+C_1}{x^2+px+q} + \frac{B_2x+C_2}{(x^2+px+q)^2} + \dots + \frac{B_nx+C_n}{(x^2+px+q)^n}$$

Step 3 - Integrate but do algebra first...

- common denominator
- equate numerator
- algebra, convert to polynomial
- equate coefficients
- find $A, B, C, \dots, n$
- If $deg(f) \geq deg(g)$ use polynomial long division

Improper Integrals

Type I: Unbounded Domain

$$\int_a^{+\infty} f(x)\,dx = \lim_{b\to+\infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{+\infty} f(x)\,dx$$

If limit exists: integral converges (finite area)
If limit DNE: integral diverges (infinite area)

Important Integral

$\int_1^{+\infty} \frac{1}{x^p}\,dx$ converges if $p>1$ diverges if $p\le 1$

Type II: Unbounded Functions

1. If f is continuous on $(a,b]$ and discontinuous at a, then
   $\int_a^b f(x)\,dx = \lim_{c\to a^+} \int_c^b f(x)\,dx \quad c>a$
2. If f is continuous on $[a,b)$ and discontinuous at b, then
   $\int_a^b f(x)\,dx = \lim_{c\to b^-} \int_a^c f(x)\,dx \quad c<b$
3. If f is cont on $[a,c)\cup(c,b]$ and discont at c, then
   $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Tests for convergence

Direct comparison test - Let f and g be continuous on $[a, +\infty)$ with $0 \le f(x) \le g(x)$ for all $x \ge a$ then

1. If $\int_a^{+\infty} g(x)\,dx$ converges, then $\int_a^{+\infty} f(x)\,dx$ converges

2. If $\int_a^{+\infty} f(x)\,dx$ diverges, then $\int_a^{+\infty} g(x)\,dx$ diverges

$\int_1^{\infty} \frac{1}{x^p}\,dx$ converges if $p>1$, diverges if $p \le 1$

Limit Comparison Test

If f and g are positive and continuous on $[a, \infty)$ and if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$, $0 < L < +\infty$ then (given over comparison) $\int_a^{\infty} f(x)\,dx$ and $\int_a^{\infty} g(x)\,dx$ both converge or both diverge

Ex. $\int_4^{+\infty} \frac{1}{\sqrt{x}+1}\,dx$ Find comp function

given $f(x) = \frac{1}{\sqrt{x}+1}$ compare to $g(x) = \frac{1}{\sqrt{x}}$

Compute limit $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\left(\frac{1}{\sqrt{x}+1}\right)}{\left(\frac{1}{\sqrt{x}}\right)} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x}+1}$ divide by $\frac{\sqrt{x}}{\sqrt{x}}$

$= \lim_{x\to\infty} \frac{1}{1+\frac{1}{\sqrt{x}}} = 1$ $L=1$, $0 < L < \infty$, test applies

Integrate comp function $\int_4^{+\infty} \frac{1}{\sqrt{x}}\,dx$ diverges $p = \frac{1}{2} \le 1$

Conclusion by LCT, $\int_4^{+\infty} \frac{1}{\sqrt{x}+1}\,dx$ diverges

Mohammad Elassaad

$\frac{47}{50}$

Math 142, Midterm 2, March 1, 2019

1. (8 points) Compute the following.

$\frac{6}{8}$

(a) $\sin^{-1}(\sqrt{3}/2) = \boxed{\frac{\pi}{3}}$ ✓

(b) $\tan^{-1}(-\sqrt{3}) = \boxed{-\frac{\pi}{3}}$ ✓

(c) $\sec^{-1}(-\sqrt{2}) = \boxed{\frac{5\pi}{4}}$ ✗

(d) $\cot(\cos^{-1}(-1/2)) = \cot\left(\frac{4\pi}{3}\right) = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{-\frac{1}{\sqrt{3}}}$ ✓

2. (3 points) Let $f(x) = x^3 + 3\sin x + 2\cos x$. Find $(f^{-1})'(2)$. $(f^{-1})'(\text{output}) = \frac{1}{f'(\text{input})}$

$\frac{3}{3}$

Output $= 2$

$\Rightarrow \quad 2 = x^3 + 3\sin x + 2\cos x$

input $= 0$ ✓

$f(x) = x^3 + 3\sin x + 2\cos x$

$f'(x) = 3x^2 + 3\cos x - 2\sin x$ ✓

✓

$\Rightarrow (f^{-1})'(2) = \frac{1}{3(0)^2 + 3\cos(0) - 2\sin(0)} = \frac{1}{0+3-0} = \boxed{\frac{1}{3}}$

8/8

$\frac{d}{dx} x^{-1} = \frac{-1}{x^2}$

3. Compute the following limits.

(a) (4 points)

$$\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) = \lim_{x\to 1}\left(\frac{(\ln x)(x)}{(\ln x)(x-1)} - \frac{(x-1)}{(\ln x)(x-1)}\right)$$

$$= \lim_{x\to 1} \frac{(\ln x)(x)-(x-1)}{(\ln x)(x-1)} \underset{LR}{=} \lim_{x\to 1} \frac{\cancel{\left(\frac{x}{x}\right)}+(\ln x)-1}{\left(\frac{1}{x}\right)(x-1)+(\ln x)} = \lim_{x\to 1}\frac{\ln x}{\left(\frac{1}{x}\right)(x-1)+(\ln x)}$$

$$\underset{LR}{=} \lim_{x\to 1} \frac{\left(\frac{1}{x}\right)}{\left(-\frac{1}{x^2}\right)(x-1)+\left(\frac{1}{x}\right)+\left(\frac{1}{x}\right)} = \boxed{\frac{1}{2}}$$

(b) (4 points)

$$\lim_{x\to 0^+} (\cos x)^{1/x^2}$$

$\Rightarrow$ indeterminate

$\Rightarrow$ logarithmic differentiation

$$\Rightarrow \lim_{x\to 0^+} \ln(\cos x)^{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{1}{x^2}\ln(\cos x)$$

$$= \lim_{x\to 0^+} \frac{\ln(\cos x)}{x^2} \underset{LR}{=} \lim_{x\to 0^+} \frac{\left(\frac{1}{\cos x}\right)(-\sin x)}{2x} = \lim_{x\to 0^+} \frac{-\tan x}{2x}$$

$$\underset{LR}{=} \lim_{x\to 0^+} \frac{-\sec^2 x}{2} = \lim_{x\to 0^+} \frac{-\left(\frac{1}{\cos x}\right)\left(\frac{1}{\cos x}\right)}{2} = -\frac{1}{2} \Rightarrow e^{-\frac{1}{2}} = \boxed{\frac{1}{\sqrt{e}}}$$

(exponentiated)

$\frac{9}{9}$

4. Compute the following derivatives. You do not need to simplify your answer.

(a) (3 points)

$$y = \frac{\log_5(\sin^{-1} x)}{x}$$

$y = (\log_5(\sin^{-1} x))(x^{-1}) \Rightarrow y' = (\log_5(\sin^{-1} x))'(x^{-1}) + (x^{-1})'(\log_5(\sin^{-1} x))$

$= \boxed{\left(\frac{1}{\sin^{-1} x}\right)\left(\frac{1}{\ln 5}\right)\left(\frac{1}{\sqrt{1-x^2}}\right)\left(\frac{1}{x}\right) - \left(\frac{1}{x^2}\right)(\log_5(\sin^{-1} x))}$

(b) (3 points)

$$y = 2^{\tan^{-1} x} + (\ln(\sin x))^{\pi}$$

$y' = \boxed{(2^{\tan^{-1} x})(\ln 2)\left(\frac{1}{1+x^2}\right) + \left(\pi(\ln(\sin x))^{\pi-1}\right)\left(\frac{\cos x}{\sin x}\right)}$

(c) (3 points)

$$y = (\ln x)^{\tan x}$$

variable to a variable
$\Rightarrow$ logarithmic differentiation

$\ln y = \ln((\ln x)^{\tan x})$

$\ln y = \tan x \ln(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = (\sec^2 x)(\ln(\ln x)) + (\tan x)\left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = \boxed{((\ln x)^{\tan x})\left((\sec^2 x)(\ln(\ln x)) + (\tan x)\left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right)\right)}$

$\frac{8}{8}$

5. Compute the following integrals.

(a) (4 points)

$$\int \frac{\tan(\ln x)}{x}\, dx$$

let $u = \ln x$ $\quad du = \frac{1}{x}$

$\Rightarrow \int \tan u \, du = \ln|\sec u| + C$

$= \boxed{\ln|\sec(\ln x)| + C}$

(b) (4 points)

$$\int \frac{2^x}{2^x+1}\, dx$$

let $u = 2^x + 1$ $\quad du = (2^x)(\ln 2) \Rightarrow \frac{1}{\ln 2} du = (2^x)$

$\frac{1}{\ln 2} \int \frac{1}{u} du = \frac{1}{\ln 2} \cdot \ln u + C$

$= \boxed{\frac{\ln|(2^x+1)|}{\ln 2} + C}$

abs value

$\frac{7}{8}$

(c) (4 points)

$$\int \frac{dx}{x\sqrt{16x^2-9}}$$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \left(\frac{1}{a}\right)\sec^{-1}\left(\frac{u}{a}\right)+C$

let $u = 4x$ $du = 4 \Rightarrow \frac{1}{4}du = 1$

$\Rightarrow \int \frac{du}{u\sqrt{u^2-3^2}} = \boxed{\left(\frac{1}{3}\right)\sec^{-1}\left|\left(\frac{4x}{3}\right)\right|+C}$

abs value

$\frac{4}{4}$

(d) (4 points)

$$\int_0^{\ln\sqrt{3}} \frac{e^x}{1+e^{2x}}\,dx$$

$\int \frac{du}{a^2+u^2} = \left(\frac{1}{a}\right)\tan^{-1}\left(\frac{u}{a}\right)$

let $u = e^x$ $du = e^x$

$\Rightarrow \int_1^{\sqrt{3}} \frac{du}{1^2+u^2} = \left(\frac{1}{1}\right)\tan^{-1}\left(\frac{e^x}{1}\right)\Big|_{\ln 1}^{\ln\sqrt{3}}$

$\boxed{\tan^{-1}(e^{\sqrt{3}}) - \tan^{-1}(e)}$

$\tan^{-1}(\sqrt{3}) - \tan^{-1}(1) = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{3}{4}$

6. (6 points) The radioactive isotope Indium-111 is often used for diagnosis and imaging in nuclear medicine. Its half life is 2.8 days. What was the initial mass of the isotope before decay, if the mass in 2 weeks was 6g?

14 days

radioactive decay $\Rightarrow$ $y = y_0 e^{-kt}$

6/6

half life = $t = \dfrac{\ln 2}{k}$

$\Rightarrow 2.8 = \dfrac{\ln 2}{k} \Rightarrow k = \dfrac{\ln 2}{2.8}$ ✓

$6 = y_0 e^{-\frac{\ln 2}{2.8}(14)}$ ✓ because $e^{-x} = \dfrac{1}{e^x}$

$\Rightarrow 6 = \dfrac{y_0}{e^{\frac{\ln 2}{2.8}(14)}}$

$\Rightarrow y_0 = \boxed{6e^{\frac{\ln 2}{2.8}(14)} \text{ grams}}$

✓

Mohammad Elassaad

16/20

Math 142, Quiz 6, March 8, 2019

Evaluate the following integrals.

1. (5 points)

$$\int \frac{\ln x}{\sqrt{x}}\,dx = \int \ln x \cdot x^{-\frac{1}{2}}\,dx$$

$$\int u\,dv = uv - \int v\,du \qquad u = \ln x \quad du = \frac{1}{x}dx$$

$$dv = x^{-\frac{1}{2}}dx \quad v = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x}$$

$$\Rightarrow \int \ln x \cdot x^{-\frac{1}{2}}dx = (\ln x)(2\sqrt{x}) - \int 2\sqrt{x}\cdot\frac{1}{x}\,dx$$

$$\Rightarrow \int \ln x \cdot x^{-\frac{1}{2}}dx = (\ln x)(2\sqrt{x}) - 2\int x^{\frac{1}{2}}\cdot x^{-1}\,dx$$

$$\Rightarrow \int \ln x \cdot x^{-\frac{1}{2}}dx = (\ln x)(2\sqrt{x}) - 2\int x^{-\frac{1}{2}}\,dx$$

$$\Rightarrow \int \ln x \cdot x^{-\frac{1}{2}}dx = (\ln x)(2\sqrt{x}) - 2\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right) + C$$

$$\Rightarrow \int \ln x \cdot x^{-\frac{1}{2}}dx = \boxed{(\ln x)(2\sqrt{x}) - (4\sqrt{x}) + C}$$

which is also...

$$2\sqrt{x}\,(\ln x - 2) + C$$

2. (5 points)

$$\int 4\cos^4 x\,dx$$

$$= 4\int (\cos^2 x)(\cos^2 x)\,dx$$

$$= 4\int \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)dx$$

$$\int \frac{1+\cos 4x}{2}\,dx$$

$$= 4\int \left(\frac{1}{2}+\frac{\cos 2x}{2}\right)\left(\frac{1}{2}+\frac{\cos 2x}{2}\right)dx$$

$$= \int \frac{1}{2}\,dx + \frac{1}{2}\int \cos 4x\,dx$$

$$= 4\int \frac{\cos 2x^2}{4} + \frac{2\cos 2x}{4} + \frac{1}{4}\,dx$$

$$= \int \cos 2x^2\,dx + 2\int \cos 2x\,dx + \int 1\,dx$$

$$\Rightarrow \frac{1}{2}x + \frac{1}{2}\left(\frac{\sin 4x}{4}\right) + \frac{2\sin 2x}{2} + x + C = \boxed{\frac{\sin 4x}{8} + \sin 2x + \frac{3}{2}x + C}$$

$$\tan^2 x = \sec^2 x - 1$$

3. (5 points)

$$\int \sec^3 x \tan^5 x\,dx$$

$$= \int (\sec x\tan x)(\sec^2 x\tan^4 x)\,dx \qquad u = \sec x$$

$$du = \sec x\tan x\,dx$$

$$= \int (\sec x\tan x)(\sec^2 x(\sec^2 x - 1)^2)\,dx$$

$$= \int u^2(u^2-1)^2\,du = \int u^2(u^4 - 2u^2 + 1)\,du$$

$$= \int u^6 - 2u^4 + u^2\,du = \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

$$= \boxed{\frac{(\sec x)^7}{7} - \frac{2(\sec x)^5}{5} + \frac{(\sec x)^3}{3} + C}$$

-1

4. (5 points)

trig sub

$\sqrt{x^2-a^2} = a\sec\theta = x$

$= 7\sec\theta = x$

$$\int \frac{\sqrt{x^2-49}}{x}\,dx, \quad x > 7$$

$dx = 7(\sec\theta\tan\theta)\,d\theta$

$\sec\theta = \frac{x}{7}$

$\cos\theta = \frac{7}{x}$

$x^2 - 49 = (7\sec\theta)^2 + 49$

$$\Rightarrow \int \frac{\sqrt{(7\sec\theta^2) + 49}}{7\sec\theta}\,7(\sec\theta\tan\theta)\,d\theta$$

$$= \int \frac{7\sec\theta + 7}{7\sec\theta}(7\sec\theta\tan\theta)\,d\theta$$

$$= \int \frac{\cancel{7\sec\theta}(7\sec\theta\tan\theta)}{\cancel{7\sec\theta}} + \frac{\cancel{7}(7\sec\theta\tan\theta)}{\cancel{7}\sec\theta}\,d\theta$$

$$= 7\int \sec\theta\tan\theta\,d\theta + 7\int \frac{\sec\theta\tan\theta}{\sec\theta}\,d\theta$$

u-sub

$u = \sec\theta$

$du = \sec\theta\tan\theta\,d\theta$

$$= 7\sec\theta + 7\int \frac{1}{u}\,du$$

$$= 7\sec\theta + 7\ln|u| + C$$

$$= 7\sec\theta + 7\ln|\sec\theta| + C$$

put back in terms of x

$\sec\theta = \frac{x}{7}$

$$= 7\left(\frac{x}{7}\right) + 7\ln\left|\frac{x}{7}\right| + C$$

$$= \boxed{x + 7\ln\left|\frac{x}{7}\right| + C}$$

-3

Mohammad Bloosood

Math 142-02

2/3/19

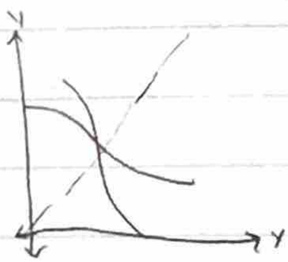
7.1 # 3, 5, 7, 11, 13, 15, 19, 29, 31, 33, 35, 37, 41, 43, 45

3. Not one to one

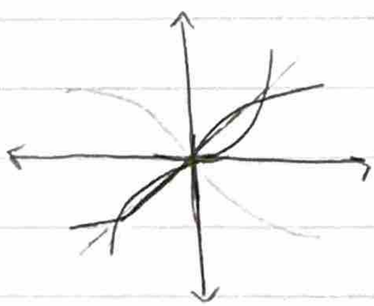
5. Yes, one to one

7. Not one to one

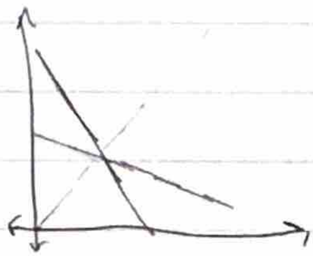
11.



13.



15.



19. $f(x) = x^2+1 \Rightarrow x = y^2+1 - \boxed{y = \sqrt{x-1} = f^{-1}(x)}$

29. $f(y) = \frac{1}{x^2} \Rightarrow x = \frac{1}{y^2} = y^2x = 1 = y^2 = \frac{1}{x} = \boxed{y = \sqrt{\frac{1}{x}} = f^{-1}(x)}$

31. $f(x) = \frac{x+3}{x-2} \Rightarrow$

$y = \frac{x+3}{x-2}$

$y(x-2) = x+3$

$xy - 2y = x+3$

$xy - x = 2y+3$

$\Rightarrow x = \frac{2y+3}{y-1}$

$\Rightarrow \boxed{y = \frac{2x+3}{x-1} = f^{-1}(x)}$

33. $f(x) = x^2-2x \Rightarrow y = x^2-2x$

$y+1 = (x-1)^2 = -\sqrt{y+1} = x-1$

$x = 1-\sqrt{y+1}$

$\boxed{f^{-1}(x) = 1-\sqrt{x+1}}$

35. $f(x) = 2x + 3 \Rightarrow f^{-1}(x) = \frac{x-3}{2}$

$2 \qquad \frac{1}{2}$

37. $f(x) = 5 - 4x \Rightarrow f^{-1}(x) = \frac{x-5}{-4}$

$-4 \qquad -\frac{1}{4}$

41. $f(x) = x^3 - 3x^2 - 1 \qquad f(3) = -1$

$f'(x) = 3x^2 - 6x$

$f'(3) = 3(3)^2 - 6(3) = 27 - 18 = 9$

$(f^{-1})'(-1) = \frac{1}{f'(3)} = \boxed{\frac{1}{9}}$

43. $f(2) = 4$

$f'(x) = \frac{1}{3}$

$f'(2) = \frac{1}{3}$

$(f^{-1})'(4) = \frac{1}{f'(2)} = \frac{1}{\frac{1}{3}} = \boxed{3}$

45. a. $f(x) = mx$

$f'(x) = m$

b. The slope is $\frac{1}{m}$

Mohammad Blossaad
2/14/19
Calc II

7.2 # 1, 3, 9, 13, 15, 17, 21, 23, 27, 29, 31, 39, 41, 43, 45, 47, 49, 55, 57, 61, 63, 67, 71, 73, 77

1. a) $\ln 0.75 = \ln \frac{3}{4} = \ln 3 - \ln 4 = \boxed{\ln 3 - 2\ln 2}$

b) $\ln \frac{4}{9} = \ln 4 - \ln 9 = \boxed{2\ln 2 - 2\ln 3}$

c) $\ln \frac{1}{2} = \ln - \ln 2 = \boxed{-\ln 2}$

d) $\ln \sqrt[3]{9} = \ln 9^{\frac{1}{3}} = \frac{1}{3}\ln 9 = \boxed{\frac{2}{3}\ln 3}$

e) $\ln 3\sqrt{2} = \ln 3 + \ln 2^{\frac{1}{2}} = \boxed{\ln 3 + \frac{1}{2}\ln 2}$

f) $\ln\sqrt{13.5} = \ln 13\frac{1}{2}^{\frac{1}{2}} = \frac{1}{2}\ln 13\frac{1}{2} = \frac{1}{2}\ln\frac{27}{2}$
$= \frac{1}{2}(\ln 27 - \ln 2) = \frac{1}{2}(\ln 3^3 - \ln 2) = \boxed{\frac{1}{2}(3\ln 3 - \ln 2)}$

3. a) $\ln\sin\theta - \ln\left(\frac{\sin\theta}{5}\right) = \ln\left(\frac{\sin\theta}{\frac{\sin\theta}{5}}\right) = \boxed{\ln 5}$

b) $\ln(3x^2 - 9x) + \ln\left(\frac{1}{3x}\right) = \boxed{\ln(x-3)}$

c) $\frac{1}{2}\ln(4t^4) - \ln 2 = \ln\sqrt{4t^4} - \ln 2 = \ln 2t^2 - 2$ ~~2 ln 2t~~
$= \ln\left(\frac{2t^2}{2}\right) = \boxed{\ln(t^2)}$

9. $y = \ln\frac{3}{x} = \ln 3x^{-1} \Rightarrow \frac{dy}{dx} = \left(\frac{1}{3x^{-1}}\right)\left(-3x^{-2}\right) = \boxed{-\frac{1}{x}}$

Back Page →

13. $y = \ln x^3 = 3\ln x \Rightarrow \frac{dy}{dx} = 3\frac{1}{x} = \boxed{\frac{3}{x}}$

15. $y = t(\ln t)^2 = \frac{dy}{dx} = (\ln t)^2 + 2t(\ln t)\cdot\frac{d}{dt}(\ln t)$

$= (\ln t)^2 + \frac{2t\ln t}{t} = \boxed{(\ln t)^2 + 2\ln t}$

17. $y = \frac{x^4}{4}\ln x - \frac{x^4}{16} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^4}{4}\ln x\right) - \frac{d}{dx}\left(\frac{x^4}{16}\right) \Rightarrow$

$\frac{4x^3}{\cancel{4}}\ln x + \frac{x^4}{4x} - \frac{4x^3}{16} = x^3\ln x + \frac{x^3}{4} - \frac{x^3}{4} = \boxed{x^3\ln x}$

21. $y = \frac{\ln x}{1+\ln x} \Rightarrow y' = \frac{vu' - uv'}{v^2} \Rightarrow \frac{(1+\ln x)(\frac{1}{x}) - (\ln x)(\frac{1}{x})}{(1+\ln x)^2}$

$= \frac{\frac{1+\ln x}{x} - \frac{\ln x}{x}}{(1+\ln x^2)} = \frac{\frac{1}{x} + \cancel{\frac{\ln x}{x}} - \cancel{\frac{\ln x}{x}}}{(1+\ln x^2)} = \boxed{\frac{1}{x(1+\ln x^2)}}$

23. $y = \ln(\ln x) \Rightarrow y' = \frac{1}{(\ln x)}\cdot\frac{1}{x} = \boxed{\frac{1}{x\ln x}}$

27. $y = \ln\left(\frac{1}{x\sqrt{x+1}}\right) = \ln\left(\frac{x^{-1}}{(x+1)^{\frac{1}{2}}}\right) \Rightarrow$

$y' = \ln x^{-1} - \ln(x+1)^{\frac{1}{2}} = -\ln x - \frac{1}{2}\ln(x+1)$

$\Rightarrow y' = -\frac{1}{x} - \frac{1}{2}\left(\frac{1}{x+1}\right) = -\frac{1}{x} - \frac{1}{2x+2} =$

$-\frac{1}{x}\cdot\frac{2+\frac{2}{x}}{2+\frac{2}{x}} - \frac{1}{2x+2} = -\frac{2+\frac{2}{x}}{2x+2} - \frac{1}{2x+2} = \boxed{\frac{-3+\frac{2}{x}}{2x+2}}$

29. $y = \frac{1+\ln t}{1-\ln t} = y' = \frac{(1-\ln t)(1+\ln t)' - (1+\ln t)(1-\ln t)'}{(1-\ln t)^2}$

$= \frac{(1-\ln t)(\frac{1}{t}) - (1+\ln t)(-\frac{1}{t})}{(1-\ln t)^2} = \frac{\frac{(1-\ln t)}{t} + \frac{(1+\ln t)}{t}}{(1-\ln t)^2}$

$= \frac{(1-\ln t) + (1+\ln t)}{t(1-\ln t^2)} = \boxed{\frac{2}{t(1-\ln t)^2}}$

31. $y = \ln(\sec(\ln\theta)) \Rightarrow y' = \frac{1}{(\sec(\ln\theta))}\cdot\frac{d}{d\theta}(\sec(\ln\theta))$

$= \frac{1}{(\sec(\ln\theta))}\cdot\frac{(\sec(\ln\theta))(\tan(\ln\theta))}{1}\cdot\frac{d}{d\theta}(\ln\theta)$

$= \frac{\tan(\ln\theta)}{1}\cdot\frac{1}{\theta} = \boxed{\frac{\tan(\ln\theta)}{\theta}}$

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Homework continued

39. $\int \frac{2y\,dy}{y^2+25} = \boxed{\ln|y^2-25|+C}$

41. $\int_0^{\pi} \frac{\sin t}{2-\cos t}\,dt = \left[\ln|2-\cos t|\right]_0^{\pi} = \ln 3 - \ln 1 = \boxed{\ln 3}$

43. $\int_1^2 \frac{2\ln x}{x}\,dx$, $u=\ln x \Rightarrow du = \frac{1}{x}\,dx \Rightarrow 2u\,du = \frac{2}{x}\,dx$

$\int_0^{\ln 2} 2u\,du = \left[\frac{2u^2}{2}\right]_0^{\ln 2} = \left[u^2\right]_0^{\ln 2} = \boxed{(\ln 2)^2}$

45. $\int_2^4 \frac{dx}{x(\ln x)^2}$, $u=\ln x \Rightarrow du = \frac{1}{x}\,dx$

$\int_{\ln 2}^{\ln 4} u^{-2}\,du = \left[\frac{u^{-1}}{-1}\right]_{\ln 2}^{\ln 4} = \left[-\frac{1}{u}\right]_{\ln 2}^{\ln 4} = -\frac{1}{\ln 4} + \frac{1}{\ln 2} = -\frac{1}{\ln 2^2} + \frac{1}{\ln 2}$

$= -\frac{1}{2\ln 2} + \frac{2}{2\ln 2} = \frac{1}{2\ln 2} = \boxed{\frac{1}{\ln 4}}$

47. $\int \frac{3\sec^2 t}{6+3\tan t}\,dt$, $u = 6+3\tan t \Rightarrow du = 3\sec^2 t\,dt$

$\int \frac{1}{u}\,du = \ln|u| + C = \boxed{\ln|6+3\tan t| + C}$

49. $\int_0^{\pi/2} \tan\frac{x}{2}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\,dx$, $u = \cos\frac{x}{2} \Rightarrow du = -\frac{1}{2}\sin\frac{x}{2}\,dx$

$\Rightarrow -2\,du = \sin\frac{x}{2}\,dx \Rightarrow -2\int_1^{\frac{\sqrt{2}}{2}} \frac{1}{u}\,du = \left[-2\ln|u|\right]_1^{\frac{\sqrt{2}}{2}}$

$\Rightarrow -2\ln\left|\frac{\sqrt{2}}{2}\right| = 2\ln\frac{2}{\sqrt{2}} = \ln\frac{2^2}{\sqrt{2}^2} = \ln\frac{4}{2} = \boxed{\ln 2}$

55. $y = \sqrt{x(x+1)} = y = (x(x+1))^{\frac{1}{2}} \Rightarrow \ln y = \frac{1}{2}\ln(x(x+1)) \Rightarrow$

$2\ln y = \ln x + \ln(x+1) \Rightarrow \frac{2y'}{y} = \frac{1}{x} + \frac{1}{x+1} \Rightarrow$

$y' = \left(\frac{1}{2}\right)\left(\sqrt{x(x+1)}\right)\left(\frac{1}{x} + \frac{1}{x+1}\right) = \left(\frac{1}{2}\right)\left(\sqrt{x(x+1)}\right)\left(\frac{(x+1)}{x(x+1)} + \frac{x}{x(x+1)}\right)$

$= \frac{\sqrt{x(x+1)}}{2} \cdot \frac{(x+1)+x}{x(x+1)} = \boxed{\frac{(x+1)\sqrt{x(x+1)} + x\sqrt{x(x+1)}}{2x(x+1)}} =$

57. $y = \sqrt{\frac{t}{t+1}} = \left(\frac{t}{t+1}\right)^{\frac{1}{2}} \Rightarrow \ln y = \frac{1}{2}\ln\left(\frac{t}{t+1}\right) =$

$2\ln y = \ln t - \ln(t+1) \Rightarrow \frac{2y'}{y} = \frac{1}{t} - \frac{1}{t+1}$

$y' = \left(\frac{1}{2}\right)\left(\frac{1}{t} - \frac{1}{t+1}\right)\left(\sqrt{\frac{t}{t+1}}\right) = \left(\frac{1}{2}\right)\left(\sqrt{\frac{t}{t+1}}\right)\left(\frac{t+1}{t(t+1)} - \frac{t}{t(t+1)}\right)$

$y' = \left(\frac{1}{2}\right)\left(\sqrt{\frac{t}{2+t}}\right)\left(\frac{1}{t(t+1)}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{t}}{\sqrt{2+t}}\right)\left(\frac{1}{t(t+1)}\right)$

$y' = \boxed{\frac{\sqrt{t}}{2(\sqrt{2+t})t(t+1)}}$

61. $y = t(t+1)(t+2) \Rightarrow \ln y = \ln t + \ln(t+1) + \ln(t+2)$

$\frac{y'}{y} = \frac{1}{t} + \frac{1}{t+1} + \frac{1}{t+2} = \quad y' = \boxed{\left(\frac{1}{t} + \frac{1}{t+1} + \frac{1}{t+2}\right)(t(t+1)(t+2))}$

63. $y = \frac{\theta+5}{\theta\cos\theta} \Rightarrow \ln y = \ln(\theta+5) - \ln(\cos\theta) - \ln(\theta)$

$\frac{y'}{y} = \frac{1}{\theta+5} - \frac{1}{\cos\theta} - \frac{1}{\theta} \Rightarrow y' = \left(\frac{1}{\theta} - \frac{1}{\theta+5} + \frac{\sin\theta}{\cos\theta}\right)\left(\frac{\theta+5}{\theta\cos\theta}\right)$

$\Rightarrow y' = \boxed{\left(\frac{1}{\theta} - \frac{1}{\theta+5} + \tan\theta\right)\left(\frac{\theta+5}{\theta\cos\theta}\right)}$

67. $y = \sqrt[3]{\frac{x(x-2)}{x^2+1}} = \ln y = \frac{1}{3}\ln\left(\frac{x(x-2)}{x^2+1}\right) = \frac{1}{3}\ln y = \ln(x(x-2)) - \ln(x^2+1)$

$\frac{3y'}{y} = \frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1} \Rightarrow y' = \boxed{\left(\frac{1}{3}\right)\left(\sqrt[3]{\frac{x(x-2)}{x^2+1}}\right)\left(\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1}\right)}$

71. $\int_1^5 (\ln 2x - \ln x)\,dx = \int_1^5 (\ln 2 + \ln x - \ln x) = \ln 2 \int_1^5 dx$

$= \ln 2\,(5-1) = \ln 2 \cdot 4 = \ln 2^4 = \boxed{\ln 16}$

73. $V = \pi\int_0^3 \left(\frac{2}{\sqrt{y+1}}\right)^2 dy = 4\pi\int_0^3 \frac{1}{y+1}\,dy = 4\pi\left[\ln|y+1|\right]_0^3$

$= 4\pi(\ln 4 - \ln 1) = \boxed{4\pi\ln 4}$

77. a) + b) =

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Section 7.3 # 1, 7, 9, 11, 13, 15, 17, 19, 21, 25, 29, 33, 37, 39, 41, 43, 47, 49, 51, 57, 59, 61, 63, 65, 69, 71, 73, 87,
89, 91, 93, 95, 99, 103, 105, 111, 113, 117

1. a) $e^{-0.3t}=27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow -0.3t \ln e = 3\ln 3 \Rightarrow -0.3t = 3\ln 3 \Rightarrow \boxed{t = -10\ln 3}$

b) $e^{kt} = \frac{1}{2} \Rightarrow kt \ln e = -\ln 2 \Rightarrow \boxed{t = -\frac{\ln 2}{k}}$

c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow (\ln 0.2)t \cancel{\ln e} = \ln 0.4 \Rightarrow \boxed{t = \frac{\ln 0.4}{\ln 0.2}}$

7. $y = e^{5-7x} \Rightarrow \frac{dy}{dx} = e^{5-7x} \cdot -7 = \boxed{-7e^{5-7x}}$

9. $y = xe^x - e^x \Rightarrow \frac{dy}{dx} = x \cdot \frac{d}{dx}e^x + \frac{d}{dx}x \cdot e^x - \frac{d}{dx}e^x = xe^x + e^x - e^x = \boxed{xe^x}$

11. $y = (x^2-2x+2)e^x \Rightarrow \frac{dy}{dx} = (2x-2)e^x + (x^2-2x+2)e^x = \cancel{2xe^x} - \cancel{2e^x} + x^2e^x - \cancel{2xe^x} + \cancel{2e^x} = \boxed{x^2e^x}$

13. $y = e^\theta(\sin\theta + \cos\theta) \Rightarrow \frac{dy}{dx} = e^\theta(\sin\theta + \cos\theta) + e^\theta(\cos\theta - \sin\theta) = \cancel{e^\theta\sin\theta} + e^\theta\cos\theta + e^\theta\cos\theta - \cancel{e^\theta\sin\theta}$
$= \boxed{2e^\theta\cos^\theta}$

15. $y = \cos(e^{-\theta^2}) \Rightarrow \frac{dy}{dx} = -\sin(e^{-\theta^2}) \cdot e^{-\theta^2} \cdot -2\theta = \boxed{2\theta e^{-\theta^2}\sin(e^{-\theta^2})}$

17. $y = \ln(3te^{-t}) = \ln 3 + \ln t + \ln e^{-t} = \ln 3 + \ln t - t \Rightarrow$
$\frac{dy}{dx} = \frac{1}{t} - 1 = \boxed{\frac{1-t}{t}}$

19. $y = \ln\left(\frac{e^\theta}{1+e^\theta}\right) = \ln e^\theta - \ln(1+e^\theta) = \theta - \ln(1+e^\theta) \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{1+e^\theta} \cdot e^\theta = 1 - \frac{e^\theta}{1+e^\theta}$
$= \boxed{\frac{1}{1+e^\theta}}$

21. $y = e^{(\cos t + \ln t)} = e^{\cos t} \cdot e^{\ln t} = e^{\cos t} \cdot t \Rightarrow \frac{dy}{dt} = e^{\cos t} + te^{\cos t} \cdot -\sin t = \boxed{(1 - t\sin t)e^{\cos t}}$

25. $\ln y = e^y \sin x \Rightarrow \frac{1}{y} y' = y' e^y \sin x + e^y \cos x \Rightarrow y'\left(\frac{1}{y} - e^y \sin x\right) = e^y \cos x$

$\Rightarrow y'\left(\frac{1 - e^y y \sin x}{y}\right) = e^y \cos x \Rightarrow y' = \boxed{\frac{e^y y \cos x}{1 - e^y y \sin x}}$

29. $\int (e^{3x} + 5e^{-x})\, dx = \int e^{3x} dx + \int 5e^{-x} dx = \boxed{\frac{e^{3x}}{3} - 5e^{-x} + C}$

33. $\int 8e^{(x+1)}\, dx = 8\int e^{(x+1)} dx = \boxed{8e^{(x+1)} + C}$

37. $\int \frac{e^{\sqrt{r}}}{\sqrt{r}} dr = \int e^{(r)^{\frac{1}{2}}} \cdot r^{(-\frac{1}{2})}$ $\quad u = r^{\frac{1}{2}} \quad du = \frac{1}{2} r^{-\frac{1}{2}} dr \quad 2du = r^{-\frac{1}{2}} dr$

$2\int e^u du = 2e^u + C = \boxed{2e^{r^{\frac{1}{2}}} + C}$

39. $\int 2t e^{-t^2} dt \quad u = -t^2 \quad du = -2t\, dt \quad -du = 2t\, dt$

$-\int e^u du = -e^u + C = \boxed{-e^{-t^2} + C}$

41. $\int \frac{e^{\frac{1}{x}}}{x^2} dx \quad u = x^{-1} \quad du = -x^{-2} dx \quad -du = x^{-2} dx$

$-\int e^u du = -e^u + C = \boxed{-e^{\frac{1}{x}} + C}$

43. $\int_0^{\frac{\pi}{4}} (1 + e^{\tan\theta}) \sec^2\theta\, d\theta \quad u = \tan\theta \quad du = \sec^2\theta\, d\theta$

$\int_0^{\frac{\pi}{4}} \sec^2\theta + \int_0^{\frac{\pi}{4}} \sec^2\theta\, e^{\tan\theta} = \int_0^{\frac{\pi}{4}} \sec^2\theta + \int_0^1 e^u du = \Big|_0^{\frac{\pi}{4}} \tan\theta + \Big|_0^1 e^u$

$= 1 - 0 + e - 1 = \boxed{e}$

47. $\int_{\ln(\frac{\pi}{6})}^{\ln(\frac{\pi}{2})} 2e^v \cos e^v\, dv \quad u = e^v \quad du = e^v dv \quad 2du = 2e^v$

$2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos u\, du = 2\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin u = 2\left(1 - \frac{1}{2}\right) = 2\left(\frac{1}{2}\right) = \boxed{1}$

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Homework / Practice Continued

49. $\int \frac{e^r}{1+e^r}\,dr \quad u = 1+e^r \quad du = e^r\,dr$

$\int \frac{1}{u}\,du = \ln u + c = \boxed{\ln(1+e^r)+c}$

51. $\frac{dy}{dt} = e^t \sin(e^t - 2)$, $y(\ln 2) = 0$

$y = \int e^t \sin(e^t-2)\,dt \quad u = e^t - 2 \quad du = e^t\,dt$

$\int \sin u\,du = -\cos u + c = -\cos(e^t-2)+c$, $y(\ln 2)=0$

$\Rightarrow -\cos(e^{\ln 2}-2)+c = 0 \Rightarrow -\cos(2-2)+c=0 \Rightarrow -\cos 0 + c = 0 \Rightarrow c = \cos 0 = 1$

$\Rightarrow \boxed{y = 1-\cos(e^t-2)}$

57. $y = 5^{\sqrt{s}} \Rightarrow \frac{dy}{dx} = 5^{\sqrt{s}} \ln 5 \, \frac{1}{2}s^{-\frac{1}{2}} = \boxed{5^{\sqrt{s}}\left(\frac{\ln 5}{2s^{\frac{1}{2}}}\right)}$

59. $y = x^{\pi} \Rightarrow y' = \boxed{\pi x^{\pi-1}}$

61. $y = (\cos\theta)^{\sqrt{2}} \Rightarrow y' = \boxed{-\sqrt{2}(\cos\theta)^{\sqrt{2}-1}(\sin\theta)}$

aren't we suppose to use $\frac{d}{dx}(a^x) = e^{x\ln a}\ln a$
$= a^x \ln a$?

* 63. $y = 7^{\sec\theta}\ln 7 \Rightarrow y' = (7^{\sec\theta}\ln 7)(\ln 7)(\sec\theta\tan\theta) = \boxed{(7^{\sec\theta})(\ln 7)^2(\sec\theta\tan\theta)}$

why can we forget about ln7

65. $y = 2^{\sin 3t} \Rightarrow y' = (2^{\sin 3t})(\ln 2)(\cos(3t)3) = \boxed{(3\cos 3t)(2^{\sin 3t})(\ln 2)}$

* 69. $y = \log_4 x + \log_4 x^2 \Rightarrow y = \frac{\ln x}{\ln 4} + \frac{\ln x^2}{\ln 4} = \frac{\ln x}{\ln 4} + 2\frac{\ln x}{\ln 4} = 3\frac{\ln x}{\ln 4}$

$\Rightarrow y' = \boxed{\frac{3}{x\ln 4}}$

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why don't we take the derivative of this?

* 71. $y = x^3 \log_{10} x = (x^3)\left(\frac{\ln x}{\ln 10}\right) = \frac{1}{\ln 10} x^3 \ln x \Rightarrow y' = \frac{1}{\ln 10}\left(x^3 \cdot \frac{1}{x} + 3x^2 \ln x\right)$

$= \left(\frac{1}{\ln 10}\right)x^2 + \left(\frac{\ln x}{\ln 10}\right)3x^2 = \boxed{\left(\frac{1}{\ln 10}\right)x^2 + 3x^2 \log x}$

73. $y = \log_3\left(\left(\frac{x+1}{x-1}\right)^{\ln 3}\right) = \frac{\ln\left(\frac{x+1}{x-1}\right)^{\ln 3}}{\ln 3} = \frac{\ln 3 \ln\left(\frac{x+1}{x-1}\right)}{\ln 3} = \ln\left(\frac{x+1}{x-1}\right) = \ln(x+1) - \ln(x-1)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x+1} - \frac{1}{x-1} = \boxed{\frac{-2}{(x+1)(x-1)}}$

87. $\int_1^{\sqrt{2}} x 2^{(x^2)} dx \qquad u = x^2 \qquad du = 2x\,dx \quad \frac{1}{2} du = x\,dx$

$\frac{1}{2}\int_1^2 2^u du = \frac{1}{2}\left[\frac{2^u}{\ln 2}\right]_1^2 = \frac{1}{2\ln 2}(2^2 - 2^1) = \frac{2}{2\ln 2} = \boxed{\frac{1}{\ln 2}}$

89. $\int_0^{\frac{\pi}{2}} 7^{\cos t} \sin t\,dt \qquad u = \cos t \qquad du = -\sin t\,dt \qquad -du = \sin t\,dt$

$-\int_1^0 7^u du = \int_0^1 7^u du = \left[\frac{7^u}{\ln 7}\right]_0^1 = \boxed{\frac{6}{\ln 7}}$

91. $\int_2^4 x^{2x}(1 + \ln x)dx \qquad u = x^{2x} \qquad \ln u = \ln x^{2x}$

$\ln u = 2x \ln x$

$\frac{1}{2}\int_{16}^{65,536} du = \frac{1}{2}[u]_{16}^{65,536} \qquad \frac{1}{u}\frac{du}{dx} = 2\ln x + 2$

$du = 2u(\ln x + 1)dx$

$= \boxed{32,760} \qquad \frac{1}{2} du = u(\ln x + 1)dx$

$\frac{1}{2} du = x^{2x}(\ln x + 1)dx$

93. $\int 3x^{\sqrt{3}} dx = 3\int x^{\sqrt{3}} dx = \boxed{3\left(\frac{x^{\sqrt{3}+1}}{\sqrt{3}+1}\right) + C}$

95. $\int_0^3 (\sqrt{2}+1)x^{\sqrt{2}} dx = (\sqrt{2}+1)\int_0^3 x^{\sqrt{2}} dx = (\sqrt{2}+1)\left[\frac{x^{\sqrt{2}+1}}{\sqrt{2}+1}\right]_0^3$

$= \cancel{(\sqrt{2}+1)}\left(\frac{3^{\sqrt{2}+1}}{\cancel{\sqrt{2}+1}}\right) = \boxed{3^{\sqrt{2}+1}}$

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* 99. $\int_1^4 \frac{\ln 2 \log_2 x}{x}\,dx = \int_1^4 \frac{\ln 2\left(\frac{\ln x}{\ln 2}\right)}{x}\,dx = \int_1^4 \frac{\ln x}{x}\,dx$

$= \left[\frac{1}{2}(\ln x)^2\right]_1^4 = \frac{1}{2}\left[(\ln 4)^2 - (\ln 1)^2\right] = \frac{1}{2}\ln 4^2 = \frac{1}{2}(2\ln 2^2) = \boxed{2(\ln 2)^2}$

* 103. $\int_0^9 \frac{2\log_{10}(x+1)}{x+1}\,dx = 2\int_0^9 \frac{\frac{\ln(x+1)}{\ln 10}}{x+1}\,dx = \frac{2}{\ln 10}\int_0^9 \frac{\ln(x+1)}{x+1}\,dx$

$= \left(\frac{2}{\ln 10}\right)\left[\frac{(\ln(x+1))^2}{2}\right]_0^9 = \left(\frac{2}{\ln 10}\right)\left(\frac{(\ln 10)^2}{2}\right) = \boxed{\ln 10}$

105. $\int \frac{dx}{x\log_{10} x} = \int\left(\frac{1}{x}\right)\left(\frac{\ln 10}{\ln x}\right)dx = \ln 10\int\left(\frac{1}{x}\right)\left(\frac{1}{\ln x}\right)dx$ $\quad u = \ln x$

$\ln 10 \int \frac{1}{u}\,du = \ln 10 \ln u + C$ $\quad du = \frac{1}{x}\,dx$

$= \boxed{(\ln 10)(\ln(\ln x)) + C}$

111. $y = (x+1)^x \Rightarrow \ln y = \ln(x+1)^x$

$\ln y = x\ln(x+1)$

$\frac{1}{y}\frac{dy}{dx} = \ln(x+1) + \frac{x}{x+1}$

$\frac{dy}{dx} = \boxed{\left(\ln(x+1) + \frac{x}{x+1}\right)(x+1)^x}$

113. $y = (\sqrt{t})^t \Rightarrow \ln y = \ln(\sqrt{t})^t \Rightarrow \frac{1}{y}\frac{dy}{dx} = t\ln(\sqrt{t})$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{t}{\sqrt{t}} + \ln\sqrt{t} \Rightarrow \frac{dy}{dx} = \boxed{(\sqrt{t})^t\left(\frac{1}{2} + \frac{\ln t}{2}\right)}$

* 117. $y = \sin(x^x) \Rightarrow y' = \cos(x^x)\cdot\frac{d}{dx}(x^x)$; if $u = x^x \Rightarrow \ln u = \ln x^x = x\ln x \Rightarrow$

$\frac{u'}{u} = x\cdot\frac{1}{x} + 1\cdot\ln x = 1 + \ln x \Rightarrow u' = x^x(1+\ln x) \Rightarrow y' = \cos x^x\cdot x^x(1+\ln x) =$

$\boxed{x^x\cos x^x(1+\ln x)}$

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Section 7.4 #23, 25, 27, 29, 31, 33, 35, 43

23. a) $y = y_0 e^{kt} \Rightarrow 0.99 y_0 = y_0 e^{k1000} = \ln 0.99 = k1000 = k = \frac{\ln 0.99}{1000} = -0.00001$

b) $0.9 y_0 = y_0 e^{-0.00001t} = \ln 0.9 = -0.00001t = \frac{\ln 0.9}{-0.00001} = t = \boxed{10{,}536}$

c) $y = y_0 e^{(-0.00001)(20{,}000)} \Rightarrow y = y_0 e^{(-0.2)} \Rightarrow y = y_0(0.82) \Rightarrow \boxed{82\%}$

25. $\frac{dy}{dt} = -0.6y \Rightarrow y = y_0 e^{-0.6t} \Rightarrow y = 100e^{-0.6(1)} \Rightarrow \boxed{54.88 \text{ grams}}$

27. $\frac{dL}{dx} = -kL \Rightarrow L = L_0 e^{-kx} \Rightarrow L = L_0 e^{-\frac{\ln 2}{18}x}$

half life $= t = \frac{\ln 2}{k} = 18 \Rightarrow k = \frac{\ln 2}{18}$

$0.1 L_0 = L_0 e^{-\frac{\ln 2}{18}x} \Rightarrow \ln 0.1 = -\frac{\ln 2}{18}x = x = -\frac{18 \ln 0.1}{\ln 2} = \boxed{59.8 \text{ ft}}$

29. $y = y_0 e^{kt}$, $y_0 = 1 \Rightarrow y = e^{kt}$, $2 = e^{k0.5} \Rightarrow \ln 2 = k0.5 = k = 2\ln 2 = \ln 4$

$y = e^{(\ln 4)(24)} = y = e^{24 \ln 4} = 4^{24} = \boxed{2.81474978 \times 10^{14}}$

31. a) $y = 10000e^{kt} \Rightarrow (0.75)(\cancel{10{,}000}) = \cancel{y_0} e^{k(1)} \Rightarrow 0.75 = e^{k} \Rightarrow k = \ln(0.75)$

$0.1 y_0 = y_0 e^{\ln(0.75)t} \Rightarrow \ln 0.1 = \ln(0.75)t \Rightarrow t = \frac{\ln 0.1}{\ln 0.75} = \boxed{8 \text{ years}}$

b) $0.0001 y_0 = y_0 e^{\ln(0.75)t} \Rightarrow \ln 0.0001 = \ln(0.75)t \Rightarrow t = \frac{\ln 0.0001}{\ln 0.75} = \boxed{32 \text{ years}}$

33. $0.9 y_0 = y_0 e^{kt} \Rightarrow 0.9 = e^{k(1)} \Rightarrow \ln 0.9 = k = \ln 0.9$

$0.2 y_0 = y_0 e^{(\ln 0.9)t} = \ln 0.2 = \ln(0.9)t = t = \frac{\ln 0.2}{\ln 0.9} = \boxed{15.28 \text{ years}}$

35. half life $= \frac{\ln 2}{k} = t \Rightarrow 24{,}360 = \frac{\ln 2}{k} \Rightarrow k = \frac{\ln 2}{24360}$

$0.2 y_0 = y_0 e^{-\frac{\ln 2}{24360}(t)} = \ln 0.2 = -\frac{\ln 2}{24360}(t) \Rightarrow t = \boxed{56562 \text{ years}}$

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7.5 # 1, 3, 5, 9, 13, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 63, 67, 73, 75, 77

1. $\lim_{x\to -2} \frac{x+2}{x^2-4} \underset{LR}{=} \lim_{x\to -2} \frac{1}{2x} = \boxed{-\frac{1}{4}}$

3. $\lim_{x\to \infty} \frac{5x^2-3x}{7x^2+1} \underset{LR}{=} \frac{10x-3}{14x} \underset{LR}{=} \boxed{\frac{10}{14}}$

5. $\lim_{x\to 0} \frac{1-\cos x}{x^2} \underset{LR}{=} \frac{\sin x}{2x} \underset{LR}{=} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

9. $\lim_{t\to -3} \frac{t^3-4t+15}{t^2-t-12} \underset{LR}{=} \frac{3t^2-4}{2t-1} = \boxed{\frac{-23}{7}}$

13. $\lim_{t\to 0} \frac{\sin t^2}{t} \underset{LR}{=} \frac{\sin t\cos t + \cos t\sin t}{1} = \boxed{0}$

17. $\lim_{\theta\to \frac{\pi}{2}} \frac{2\theta-\pi}{\cos(2\pi-\theta)} \underset{LR}{=} \frac{2}{-\sin(2\pi-\theta)(-1)} = \boxed{\frac{2}{-1}}$

19. $\lim_{\theta\to \frac{\pi}{2}} \frac{1-\sin\theta}{1+\cos 2\theta} \underset{LR}{=} \frac{-\cos\theta}{-\sin(2\theta)\cdot 2} \underset{LR}{=} \frac{\sin\theta}{-2\cdot\cos(2\theta)\cdot 2} = \boxed{\frac{1}{4}}$

21. $\lim_{x\to 0} \frac{x^2}{\ln(\sec x)} \underset{LR}{=} \frac{2x}{\cos x\cdot\sec x\tan x} \underset{LR}{=} \frac{2}{-\sin x\cdot\sec x\tan x + \cos x\cdot(\sec x\tan x\cdot\tan x + \sec x\sec^2 x)} = \boxed{2}$

23. $\lim_{t\to 0} \frac{t(1-\cos t)}{t-\sin t} \underset{LR}{=} \frac{(1-\cos t)+t\cdot\sin t}{1-\cos t} \underset{LR}{=} \frac{\sin t+\sin t+t\cos t}{\sin t} \underset{LR}{=} \frac{\cos t+\cos t+\cos t-t\sin t}{\cos t} = \boxed{3}$

25. $\lim_{x\to (\frac{\pi}{2})^-} \left(x-\frac{\pi}{2}\right)\sec x = \frac{(x-\frac{\pi}{2})}{\cos x} \underset{LR}{=} \frac{1}{-\sin x} = \boxed{-1}$

27. $\lim_{\theta\to 0} \frac{3^{\sin\theta}-1}{\theta} \underset{LR}{=} \frac{3^{\sin\theta}\cdot\ln 3\cdot\cos\theta}{1} = \boxed{\ln 3}$

29. $\lim_{x\to 0} \frac{x2^x}{2^x-1} \underset{LR}{=} \frac{2^x+x(2^x\ln 2)}{2^x\ln 2} = \boxed{\frac{1}{\ln 2}}$

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31. $\lim_{x\to\infty} \frac{\ln(x+1)}{\log_2 x} = \lim_{x\to\infty} \frac{\ln(x+1)}{\frac{\ln x}{\ln 2}} = (\ln 2)\lim_{x\to\infty} \frac{\ln(x+1)}{\ln x} \underset{LR}{=} (\ln 2)\lim_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x}} = (\ln 2)\lim_{x\to\infty} \frac{x}{x+1} \underset{LR}{=}$

$\ln 2 \lim_{x\to\infty} \frac{1}{1} = \ln 2\,(1) = \boxed{\ln 2}$

33. $\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x} \underset{LR}{=} \frac{\frac{2x+2}{x^2+2x}}{\frac{1}{x}} = \frac{2x^2+2x}{x^2+2x} \underset{LR}{=} \frac{4x+2}{2x+2} = \boxed{1}$

35. $\lim_{y\to 0} \frac{\sqrt{5y+25}-5}{y} \underset{LR}{=} \frac{(\frac{1}{2})(5y+25)^{-\frac{1}{2}}\cdot 5}{1} = \frac{5}{2\sqrt{5y+25}} = \frac{5}{10} = \boxed{\frac{1}{2}}$

37. $\lim_{x\to\infty} (\ln 2x - \ln(x+1)) = \lim_{x\to\infty} \ln\left(\frac{2x}{x+1}\right) = \ln\left(\lim_{x\to\infty} \frac{2x}{x+1}\right) \underset{LR}{=} \boxed{\ln 2}$

39. $\lim_{x\to 0^+} \frac{(\ln x)^2}{\ln(\sin x)} \underset{LR}{=} \frac{\frac{2\ln x}{x}}{\frac{\cos x}{\sin x}} = \frac{2\ln x \sin x}{x\cos x} = \frac{2\ln x}{\cos x}\cdot\frac{\sin x}{x} = -\infty\cdot 1 = \boxed{-\infty}$

41. $\lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{1}{\ln x}\right) = \frac{\ln x - (x-1)}{(x-1)\ln x} \underset{LR}{=} \frac{\frac{1}{x}-1}{\frac{1}{x}(x-1)+\ln x} = \frac{1-x}{(x-1)+x\ln x} \underset{LR}{=}$

$\frac{-1}{1+\frac{x}{x}+\ln x} = \frac{-1}{1+1+0} = \boxed{-\frac{1}{2}}$

43. $\lim_{\theta\to 0} \frac{\cos\theta - 1}{e^\theta - \theta - 1} \underset{LR}{=} \frac{-\sin\theta}{e^\theta - 1} \underset{LR}{=} \frac{-\cos\theta}{e^\theta} = \boxed{-1}$

45. $\lim_{t\to\infty} \frac{e^t + t^2}{e^t - t} \underset{LR}{=} \frac{e^t + 2t}{e^t - 1} \underset{LR}{=} \frac{e^t + 2}{e^t} \underset{LR}{=} \frac{e^t}{e^t} = \boxed{1}$

47. $\lim_{x\to 0} \frac{x - \sin x}{x\tan x} \underset{LR}{=} \frac{1-\cos x}{x\sec^2 x + \tan x} \underset{LR}{=} \frac{\sin x}{x\tan x + \sec^2 x + \sec^2 x} = \boxed{0}$

49. $\lim_{\theta\to 0} \frac{\theta - \sin\theta\cos\theta}{\tan\theta - \theta} \underset{LR}{=} \frac{1 + \sin^2\theta - \cos^2\theta}{\sec^2\theta - 1} = \frac{2\sin^2\theta}{\tan^2\theta} = 2\cos\theta^2 = \boxed{2}$

51. $\lim_{x\to 1^+} x^{\frac{1}{1-x}} = \left(\frac{1}{1-x}\right)\ln x = \frac{\ln x}{1-x} \underset{LR}{=} \frac{\frac{1}{x}}{-1} = -1 = e^{-1} = \boxed{\frac{1}{e}}$

$1^\infty \quad \infty\cdot 0$

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7.5 continued

53. $\lim_{x\to\infty}(\ln x)^{\frac{1}{x}} = \frac{1}{x}\ln x = \frac{\ln x}{x} \underset{LR}{=} \frac{\frac{1}{x}}{1} = 0 = e^0 = \boxed{1}$

57. $\lim_{x\to\infty}(1+2x)^{\frac{1}{(2\ln x)}} = \left(\frac{1}{2\ln x}\right)\ln(1+2x) = \frac{\ln(1+2x)}{2\ln x} \underset{LR}{=} \frac{\frac{2}{1+2x}}{\frac{2}{x}} = \frac{2x}{2+4x} \underset{LR}{=} \frac{2}{4} = \frac{1}{2} = \boxed{e^{\frac{1}{2}}}$

55. $\lim_{x\to0^+} x^{-\frac{1}{\ln x}} = -\frac{1}{\ln x}\ln x = \frac{-\ln x}{\ln x} = -1 = e^{-1} = \boxed{\frac{1}{e}}$

59. $\lim_{x\to0^+} x^{x} = x\ln x = \frac{\ln x}{\frac{1}{x}} \underset{LR}{=} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -\frac{x^2}{x} = -x = 0 = e^0 = \boxed{1}$

63. $\lim_{x\to0} x^2\ln x = \frac{\ln x}{x^{-2}} \underset{LR}{=} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \frac{x^3}{-2x} \underset{LR}{=} \frac{3x^2}{-2} = \boxed{0}$

67. $\lim_{x\to\infty}\frac{\sqrt{9x+1}}{\sqrt{x+1}} = \sqrt{\lim_{x\to\infty}\frac{9x+1}{x+1}} = \sqrt{\frac{9}{1}} = \boxed{3}$

73. $\lim_{x\to\infty}\frac{e^{x^2}}{xe^x} = \frac{e^x\cancel{e^x}}{x\cancel{e^x}} = \frac{e^x}{x} \underset{LR}{=} \frac{e^x}{1} = \boxed{\infty}$

75. a is wrong because the situation is not $\frac{0}{0}$ or $\frac{\infty}{\infty}$ therefore you cannot do L'Hopitals rule, b is correct, you just need to plug in the value

77. a is wrong, $0\cdot(-\infty)$ = indeterminate form
b is wrong, $0\cdot(-\infty)$ = indeterminate form
c is wrong, $\frac{-\infty}{\infty}$ ≠ indeterminate form
d is correct, correct use of L'Hopitals Rule

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7.6 ✱ 1,3,5,7,9,11,23,27,29,31,33,39,43,45,47,55,59,61,63,65,67,71,75,81,83,85,87,91,93,95

1. a) $\tan^{-1}(1) = \boxed{\frac{\pi}{4}}$ b) $\tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$ c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$

3. a) $\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$ b) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\pi}{4}}$ c) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{-\frac{\pi}{3}}$

5. a) $\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$ b) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \boxed{\frac{3\pi}{4}}$ c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{6}}$

7. a) $\sec^{-1}(-\sqrt{2}) = \boxed{\frac{3\pi}{4}}$ b) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \boxed{\frac{\pi}{6}}$ c) $\sec^{-1}(-2) = \boxed{\frac{2\pi}{3}}$

9. $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \sin\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

11. $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right) = \boxed{-\frac{1}{\sqrt{3}}}$

23. $y = \sin^{-1}\sqrt{2}t = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2} = \boxed{\frac{\sqrt{2}}{\sqrt{1-2t^2}}}$

27. $y = \csc^{-1}(x^2+1) = -\frac{1}{(x^2+1)\sqrt{(x^2+1)^2-1}} \cdot 2x = \boxed{-\frac{2x}{(x^2+1)\sqrt{(x^2+1)^2-1}}}$

✱ 29. $y = \sec^{-1}\left(\frac{1}{t}\right) = \frac{1}{\left(\frac{1}{t}\right)\sqrt{\left(\frac{1}{t}\right)^2-1}} \cdot \frac{1}{-t^2} = \boxed{-\frac{1}{t\sqrt{\left(\frac{1}{t^2}\right)-1}}}$

book says $= \cos^{-1}(t) = \boxed{-\frac{1}{\sqrt{1-t^2}}}$

31. $y = \cot^{-1}\sqrt{t} = -\frac{1}{1+\sqrt{t}^2} \cdot \frac{1}{2}t^{-\frac{1}{2}} = \boxed{-\frac{1}{2\sqrt{t}(1+t)}}$

33. $y = \ln(\tan^{-1}x) = \frac{1}{\tan^{-1}x} \cdot \frac{1}{1+x^2} = \boxed{\frac{1}{(1+x^2)(\tan^{-1}x)}}$

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39. $\tan^{-1}\sqrt{x^2-1} + \csc^{-1} x = \frac{\left(\frac{1}{2}\right)(x^2-1)^{-\frac{1}{2}}(2x)}{1+\sqrt{x^2-1}^2} - \frac{1}{|x|\sqrt{x^2-1}}$

$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{|x|\sqrt{x^2-1}} = \boxed{0 \text{ for } x>1}$

43. $\int \frac{dx}{\sqrt{9-x^2}} = \boxed{\sin^{-1}\left(\frac{x}{3}\right)+C}$

45. $\int \frac{dx}{17+x^2} = \int \frac{dx\ 1}{(\sqrt{17})^2+x^2} = \boxed{\frac{1}{\sqrt{17}}\tan^{-1}\left(\frac{x}{\sqrt{17}}\right)+C}$

47. $\int \frac{dx}{x\sqrt{25x^2-2}}$ $\quad u=5x \quad du=5\,dx$

$\int \frac{du}{u\sqrt{u^2-2}} = \boxed{\frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{5x}{\sqrt{2}}\right|+C}$

55. $\int \frac{3\,dr}{\sqrt{1-4(r-1)^2}}$ $\quad u=2(r-1) \quad du=2dr \quad \frac{1}{2}du=dr$

$\frac{3}{2}\int \frac{du}{\sqrt{1-u^2}} = \boxed{\frac{3}{2}\sin^{-1}(2(r-1))+C}$

59. $\int \frac{dx}{(2x-1)\sqrt{(2x-1)^2-4}}$ $\quad u=2x-1 \quad du=2dx \quad \frac{1}{2}du=dx$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-2^2}} = \boxed{\frac{1}{4}\sec^{-1}\left(\frac{2x-1}{2}\right)+C}$

61. $\int_{-\pi/2}^{\pi/2} \frac{2\cos\theta\,d\theta}{1+(\sin\theta)^2} = 2\int_{-\pi/2}^{\pi/2} \frac{\cos\theta\,d\theta}{1+(\sin\theta)^2}$ $\quad u=\sin\theta \quad du=\cos\theta\,d\theta$

$=2\int_{-1}^{1} \frac{du}{1+u^2} = 2\left(\frac{1}{1}\tan^{-1}(u)\Big|_{-1}^{1}\right) = 2\left(\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right) = \frac{2}{1}\cdot\frac{2\pi}{4} = \boxed{\pi}$

63. $\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$ $\quad u=e^x \quad du=e^x dx$

$\int_1^{\sqrt{3}} \frac{du}{1+u^2} = \tan^{-1}(u)\Big|_1^{\sqrt{3}} = \frac{\pi}{3}-\frac{\pi}{4} = \frac{4\pi}{12}-\frac{3\pi}{12} = \boxed{\frac{\pi}{12}}$

65. $\int \frac{y\,dy}{\sqrt{1-y^4}}$ $\quad u=y^2 \quad du=2y\,dy \quad \frac{1}{2}du=y\,dy$

$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \boxed{\frac{1}{2}\sin^{-1}(y^2)+C}$

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7.6 Continued

67. $\int \frac{dx}{\sqrt{-x^2+4x-3}} \Rightarrow -x^2+4x-3 \Rightarrow x^2-4x+3 \Rightarrow x^2-4x+4=-3+4 \Rightarrow 1-(x-2)^2$

$\int \frac{dx}{\sqrt{1-(x-2)^2}} = \boxed{\sin^{-1}(x-2)+C}$

71. $\int \frac{dy}{y^2-2y+5} \Rightarrow y^2-2y+5 \Rightarrow y^2-2y+1=-5+1 \Rightarrow (y-1)^2+4$

$\int \frac{dy}{(y-1)^2+2^2} = \boxed{\frac{1}{2}\tan^{-1}\left(\frac{y-1}{2}\right)+C}$

75. $\int \frac{x+4}{x^2+4}\,dx = \int \frac{x}{x^2+4}\,dx + 4\int \frac{1}{x^2+4}\,dx \rightarrow 4\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$

$u=x^2+4 \quad du=2x\,dx \quad \frac{1}{2}du=x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln u \Rightarrow \boxed{\frac{1}{2}\ln(x^2+4)+2\tan^{-1}\left(\frac{x}{2}\right)+C}$

81. $\int \frac{e^{\sin^{-1}x}\,dx}{\sqrt{1-x^2}} \quad u=\sin^{-1}x \quad du=\frac{1}{\sqrt{1-x^2}}\,dx$

$\int e^u\,du = e^u+C = \boxed{e^{\sin^{-1}x}+C}$

83. $\int \frac{(\sin^{-1}x)^2\,dx}{\sqrt{1-x^2}} \quad u=\sin^{-1}x \quad du=\frac{1}{\sqrt{1-x^2}}\,dx$

$\int u^2\,du = \frac{u^3}{3}+C = \boxed{\frac{(\sin^{-1}x)^3}{3}+C}$

85. $\int \frac{dy}{(\tan^{-1}y)(1+y^2)} \quad u=\tan^{-1}y \quad du=\frac{1}{1+y^2}\,dy$

$\int \frac{1}{u} = \ln|u|+C = \boxed{\ln|\tan^{-1}y|+C}$

87. $\int_{\sqrt{2}}^{2} \frac{\sec^2(\sec^{-1}x)\,dx}{x\sqrt{x^2-1}} \quad u=\sec^{-1}x \quad du=\frac{1}{x\sqrt{x^2-1}}$

$\int_{\pi/4}^{\pi/3}\sec^2 u\,du = \tan u\Big|_{\pi/4}^{\pi/3} = \boxed{\sqrt{3}-1}$

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91. $\lim\limits_{x\to 0} \frac{\sin^{-1} 5x}{x} \underset{LR}{=} \lim\limits_{x\to 0} \frac{\frac{5}{\sqrt{1-5x^2}}}{1} = \lim\limits_{x\to 0} \frac{5}{\sqrt{1-5x^2}} = \boxed{5}$

93. $\lim\limits_{x\to\infty} x \tan^{-1} \frac{2}{x} = \lim\limits_{x\to\infty} \frac{\tan^{-1}\left(\frac{2}{x}\right)}{\frac{1}{x}} \underset{LR}{=} \lim\limits_{x\to\infty} \frac{\frac{-\frac{2}{x^2}}{\sqrt{1+\left(\frac{2}{x}\right)^2}}}{-\frac{1}{x^2}} = \lim\limits_{x\to\infty} \frac{(2)\cancel{\left(-\frac{1}{x^2}\right)}\cancel{\left(-x^2\right)}}{\sqrt{1+\left(\frac{2}{x}\right)^2}} = \boxed{2}$

95. $\lim\limits_{x\to 0} \frac{\tan^{-1} x^2}{x\sin^{-1} x} \underset{LR}{=} \lim\limits_{x\to 0} \frac{\frac{2x}{1+x^4}}{\sin^{-1}x + \frac{x}{\sqrt{1-x^2}}} = \lim\limits_{x\to 0} \frac{2x\left(\sin^{-1}x + \frac{\sqrt{1-x^2}}{x}\right)}{1+x^4} = \boxed{0}$

Mohammad Elassaad

Math 142, Quiz 5, February 22, 2019

19/20

Compute the limits.

1. (4 points)

$$\lim_{x\to 0} \frac{\overset{\nearrow 0}{x \sin x}}{\underset{\searrow 0}{1-\cos x}}$$

$$\lim_{x\to 0} \frac{\overset{\nearrow 0}{x\sin x}}{\underset{\searrow 0}{1-\cos x}} \underset{LR}{=} \lim_{x\to 0} \frac{\overset{\nearrow 0}{\sin x + x\cos x}}{\underset{\searrow 0}{\sin x}} \underset{LR}{=} \lim_{x\to 0} \frac{\overset{\nearrow 2}{\cos x - x\sin x + \cos x}}{\underset{\searrow 1}{\cos x}} = \frac{2}{1} = 2$$

$$\boxed{2}$$

2. (4 points)

$$\lim_{x\to 0^+} \frac{\overset{\nearrow \infty}{\ln(e^x-1)}}{\underset{\searrow -\infty}{\ln x}}$$

$$\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} \underset{LR}{=} \lim_{x\to 0^+} \frac{\frac{1}{e^x-1}\cdot e^x}{\frac{1}{x}} = \lim_{x\to 0} \frac{\overset{\nearrow 0}{x e^x}}{\underset{\searrow 0}{e^x-1}}$$

$$\underset{LR}{=} \lim_{x\to 0} \frac{\overset{\nearrow 1}{e^x + xe^x}}{\underset{\searrow 1}{e^x}} = \frac{1}{1} = \boxed{1}$$

3. (4 points)

$$\lim_{x\to+\infty} x\tan(1/x)$$

$$\lim_{x\to+\infty} x \tan(1/x) = \lim_{x\to+\infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} \underset{LR}{=} \lim_{x\to+\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{-\frac{1}{x^2}}}{\cancel{-\frac{1}{x^2}}}$$

$$= \lim_{x\to+\infty} \sec^2\left(\frac{1}{x}\right) = \lim_{x\to+\infty} \frac{1}{\cos\left(\frac{1}{x}\right)}\cdot\frac{1}{\cos\left(\frac{1}{x}\right)} = \frac{1}{1} = \boxed{1}$$

4. (4 points)

$$\lim_{x\to 0^+}\left(\frac{3x+1}{x} - \frac{1}{\sin x}\right)$$

$$\lim_{x\to 0^+}\left(\frac{3x+1}{x} - \frac{1}{\sin x}\right) = \lim_{x\to 0^+}\left(\frac{(3x+1)(\sin x)}{x\sin x} - \frac{x}{x\sin x}\right)$$

$$= \lim_{x\to 0^+} \frac{(3x+1)(\sin x) - x}{x\sin x} \underset{LR}{=} \lim_{x\to 0} \frac{3\sin x + (3x+1)\cos x - 1}{x\cos x + \sin x}$$

$$= \lim_{x\to 0^+} \frac{3\sin x + 3x\cos x + \cos x - 1}{x\cos x + \sin x} \underset{LR}{=} \lim_{x\to 0^+} \frac{3\cos x + 3\cos x - 3x\sin x - \sin x}{\cos x - x\sin x + \cos x}$$

$$= \frac{6}{2} = \boxed{3}$$

5. (4 points)

$$\lim_{x\to 0} \underset{\downarrow \; 1}{(e^x + x)}^{\overset{\nearrow \infty}{1/x}} \Rightarrow 1^{\infty} = \text{indeterminate}$$

$$\lim_{x \to 0} (e^x + x)^{\frac{1}{x}} \Rightarrow \lim_{x \to 0} \ln(e^x + x)^{\frac{1}{x}} = \lim_{x \to 0} \underset{\downarrow \; \infty}{\frac{1}{x}} \underset{\downarrow \; 0}{\ln(e^x + x)}$$

$$= \lim_{x \to 0} \frac{\overset{\nearrow 0}{\ln(e^x + x)}}{\underset{\downarrow \; 0}{x}} \overset{LR}{=} \lim_{x \to 0} \frac{\frac{1}{e^x + x} \cdot e^x + 1}{1}$$

$$= \lim_{x \to 0} \frac{\overset{\nearrow 2}{e^x + 1}}{\underset{\downarrow \; 1}{e^x + x}} = \frac{2}{1} = \boxed{2}$$

$\downarrow$

e^2

-1

Mohammad Elassaad

1/18/19

Math 142, Quiz 1, January 18, 2019

$\frac{18}{20}$

1. Compute the following integrals.

(a) (4 points)

$$\int \frac{\sin(2x+1)}{\cos^2(2x+1)}\,dx$$

$\int \frac{\sin(2x+1)}{\cos^2(2x+1)}dx = \int \frac{\sin(2x+1)}{(\cos(2x+1))^2}dx$

let $u = \cos(2x+1)$

$du = -\sin(2x+1)\cdot 2\,dx$

$-\frac{1}{2}du = \sin(2x+1)\,dx$

$\Rightarrow -\frac{1}{2}\int \frac{1}{u^2}du = -\frac{1}{2}\int u^{-2}du$

$= -\frac{1}{2}\cdot\frac{u^{-1}}{-1} + C = -\frac{1}{2}\cdot -\frac{1}{u} + C = \frac{1}{2u} + C$

$= \boxed{\frac{1}{2\cos(2x+1)} + C}$

(b) (4 points)

$$\int_1^4 \frac{10\sqrt{x}}{(1+x^{3/2})^2}\,dx$$

$\int_1^4 \frac{10\sqrt{x}}{(1+x^{3/2})^2}dx =$

let $u = 1 + x^{3/2}$

$du = \frac{3}{2}x^{\frac{1}{2}}dx$

$\frac{2}{3}du = \sqrt{x}\,dx$

$\frac{20}{3}du = 10\sqrt{x}\,dx$

upper bound $= 1+\sqrt{4^3} = 9$

lower bound $= 1+\sqrt{1^3} = 2$

$\frac{20}{3}\int_2^9 \frac{1}{u^2}du = \frac{20}{3}\int_2^9 u^{-2}du$

$= \frac{20}{3}\Big|_2^9 \frac{u^{-1}}{-1} = \frac{20}{3}\Big|_2^9 -\frac{1}{u} =$

$\frac{20}{3}\left(-\frac{1}{9}+\frac{1}{2}\right) = \frac{20}{3}\left(-\frac{2}{18}+\frac{9}{18}\right)$

$= \frac{20}{3}\left(\frac{7}{18}\right) = \frac{140}{54} = \boxed{\frac{70}{27}}$

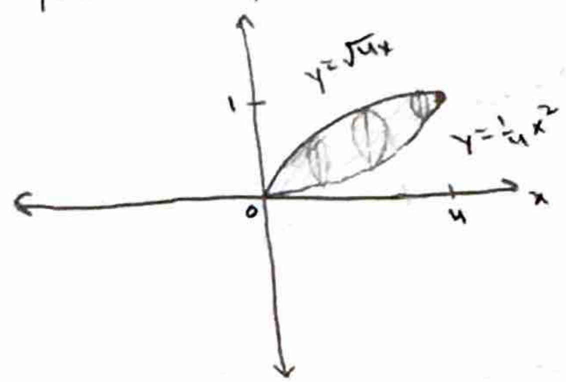
$18 \times 3 = 54$

2. (4 points) Set up, but **do not evaluate**, the integral to compute the volume of the following solid. The solid lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $x^2 = 4y$ to the parabola $y^2 = 4x$.

$y = \sqrt{4x}$

Disk Method

$y = \frac{1}{4}x^2$



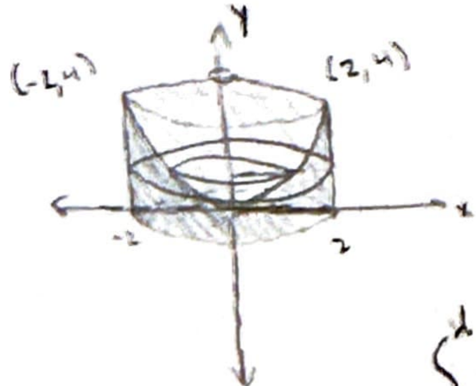
$$\int_a^b A(x)\,dx = \int_a^b \pi (R(x))^2\,dx$$

$$R = \frac{\sqrt{4y} - \frac{1}{4}x^2}{2}$$

$$\boxed{\int_0^4 \pi \left(\frac{\sqrt{4x} - \frac{1}{4}x^2}{2}\right)^2 dx}$$

3. Let R be the region bounded by $y = x^2$, $y = 0$, $x = 2$. Set up, **but do not evaluate**, the integral to the compute the volume of the solid obtained by revolving R about:

(a) (4 points) the y-axis



Washer Method

$y = x^2 \quad = \quad x = \sqrt{y}$

$R = 2 - \sqrt{y}$

$r = \sqrt{y} - 0 = \sqrt{y}$

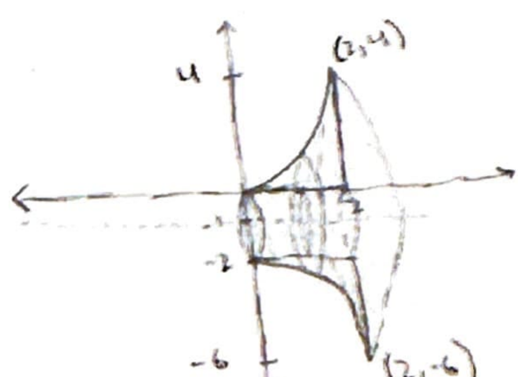
$\int_c^d \pi (R(y))^2 - \pi (r(y))^2$

$\boxed{\int_0^4 \pi (2-\sqrt{y})^2 - \pi (\sqrt{y})^2} \; dy$

2

−1

(b) (4 points) the line $y = -1$



Washer Method

$y = x^2$

$R = x^2 - (-1) = x^2 + 1$

$r = 0 - (-1) = 1$

$\int_a^b \pi (R(x))^2 - \pi (r(x))^2$

$\int_0^2 \pi (x^2+1)^2 - \pi (1)^2 = \boxed{\int_0^2 \pi (x^2+1)^2 - \pi} \; dx$

−1

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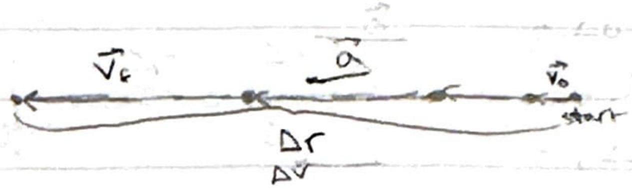
Graph all except circled

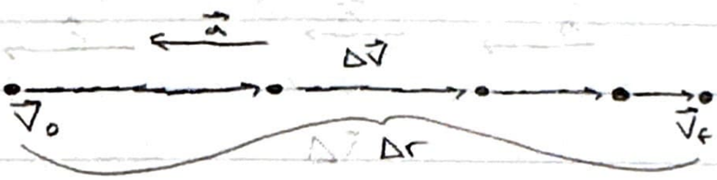
Chapter 1 # 6, 7, 8, 18, 37, 41 + Ch. 2 # 2, 6, 12, 17, 19, 22, 34, 37, 48, 67, 86

Ch. 1

6.

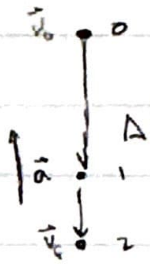


7.



8. a) The average speed between points 1 and 2 is less than 0 and one because the points are closer.

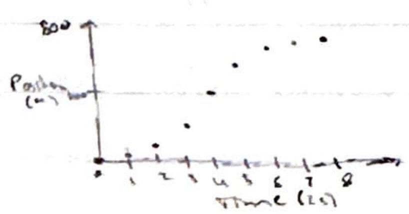
b)



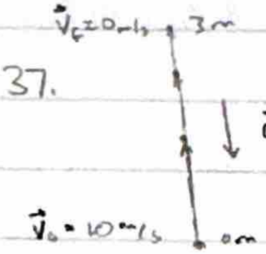
18. a)

Time t (2sec)	Position x (m)
0	0
1	25
2	100
3	200
4	400
5	520
6	600
7	670
8	720

b)



37.



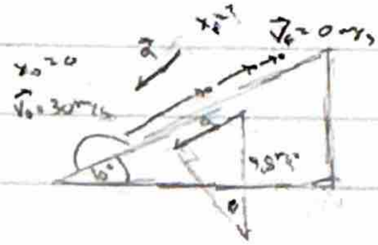
$v_{fy} = v_{0y} - g\Delta t$

$0\,m/s = 10\,m/s - 9.8\,m/s^2\,\Delta t$

$-10\,m/s = -9.8\,m/s^2\,\Delta t$

$\boxed{1.02\,s = \Delta t}$

41.



$\sin\theta = \frac{a}{9.8\,m/s^2}$ $-9.8\,m/s^2 \sin 10° = a = -1.7\,m/s^2$

$v_{fx}^2 = v_{0x}^2 + 2a(x_f - x_0)$

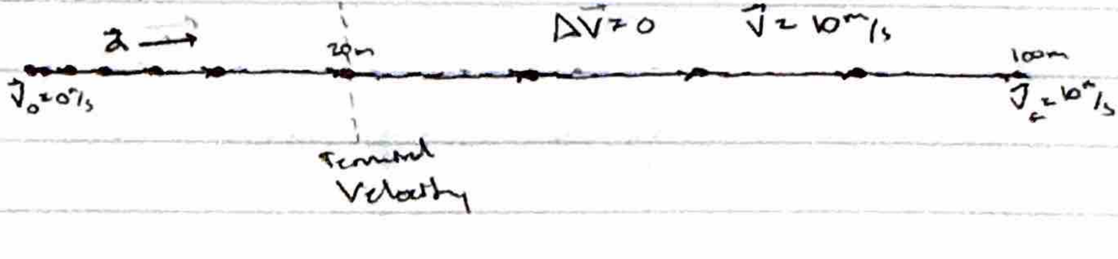
$0\,m/s^2 = 30\,m/s^2 - 2(1.7\,m/s^2)(x_f - 0)$

$-900 = -3.4(x_f) = \boxed{264\text{ meters}}$

Ch.2

DONT CORRECT THESE... WRONG PROBLEMS!!!

2. An rc car starts moving forward. It accelerates to terminal velocity in 6 seconds. It maintains a speed of 10 m/s for 4 more seconds.



6. a) B b) D c) A, G, E d) No

12. a) $a > g$, the ball has an upwards velocity

b) $a = g$, the ball has no velocity

c) $a < g$, the ball has a downwards velocity

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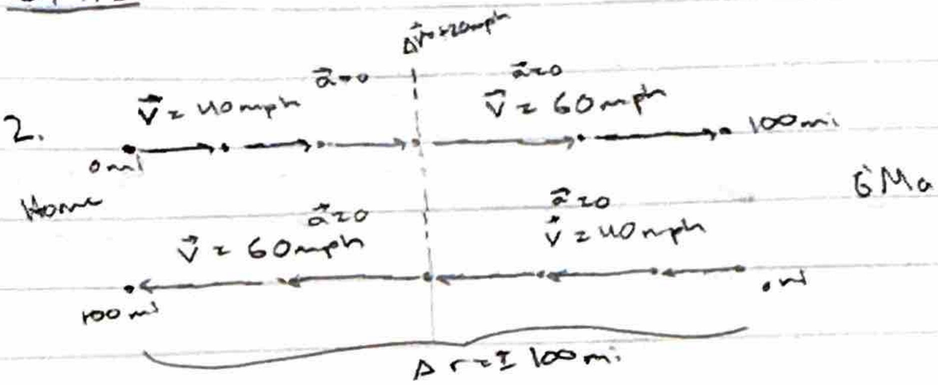
Homework continued

Ch. 2

2.



a) $\frac{40mph + 60mph}{2} = \boxed{50mph}$

b) $\boxed{\text{50mph, or -50mph in relation to first diagram}}$

6. a) Yes, $t = 1$ sec b/c $m/s = 0$

b) $x_f = x_0 + v_{0x}\Delta t + \frac{1}{2} a_x (\Delta t)^2$

$x_f = 10m + -4 m/s (2s) + \frac{1}{2} 4 m/s^2 (2s)^2 = \boxed{10m}$

$x_f = 10m + -4 m/s (4s) + \frac{1}{2} 4 m/s^2 (4s)^2 = \boxed{26m}$

12. a) $\boxed{\vec{v}_1 = 4 m/s}$ $\boxed{\vec{a}_1 = 0 m/s^2}$

$x_f = x_0 + v_{0x}\Delta t + \frac{1}{2} a_x (\Delta t)^2$

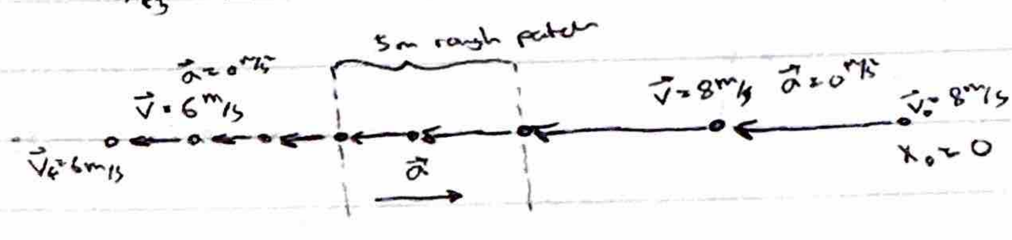
$x_f = 2m + 4 m/s (1s) + \cancel{\frac{1}{2}(0 m/s^2)(1s)^2} = \boxed{6m}$

b) $\boxed{\vec{v}_3 = -2 m/s}$ $\boxed{\vec{a}_3 = -2 m/s^2}$

$x_{f2} = 2m + 4 m/s (2s) + \cancel{\frac{1}{2}(0 m/s^2)(2s)^2} = 10m$

$x_{f3} = 10m + 4 m/s (1s) + \frac{1}{2}(-2 m/s^2)(1s)^2 = \boxed{13m}$

17.



17. (continued) $V_{fx}^2 = V_{0x}^2 + 2a(x_f - x_0)$

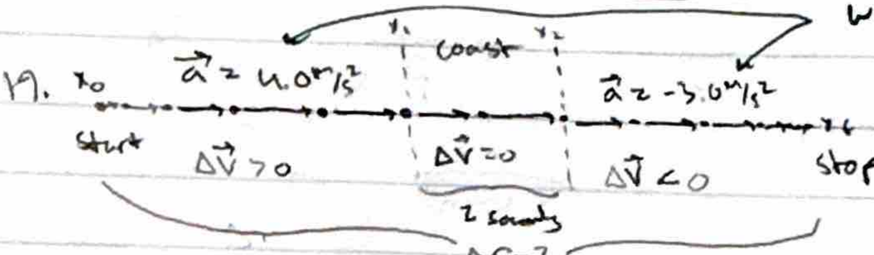
$(6\,m/s)^2 = (8\,m/s)^2 + 2a(5m - 0m)$

$36\,m^2/s^2 = 64\,m^2/s^2 + a\,10m$

$\boxed{a = -2.8\,m/s^2}$

write in $\vec{a}$ vectors (-1)

19.



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Acceleration, $x_1 = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$

$x_1 = 0m + 0\,m/s(6s) + \frac{1}{2}(4.0\,m/s^2)(6s)^2 = 72\,m$

$V_{1x} = V_{0x} + a_x\Delta t$

$V_{1x} = 0\,m/s + (4\,m/s^2)(6\,sec) = 24\,m/s$

Coast, $x_2 = x_1 + v_1\Delta t_2 + \frac{1}{2}a(\Delta t)^2$

$x_2 = 72m + 24\,m/s(2s) + \frac{1}{2}(0)(2s)^2 = 120\,m$

Deceleration, $x_f = x_2 + V_2\Delta t + \frac{1}{2}a(\Delta t)^2$

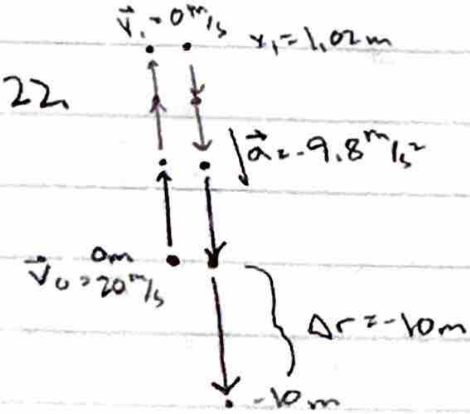
$x_f = 120m + 24\,m/s\,\Delta t + \frac{1}{2}(-3\,m/s^2)(\Delta t)^2$

$V_{fx}^2 = V_{x2}^2 + 2a(x_f - x_2)$

$(6\,m/s)^2 = (24\,m/s)^2 + 2(-3\,m/s^2)(x_f - 120m)$

$\boxed{x_f = 216\,m}$

22.



a) $V_1^2 = V_0^2 + 2a(x_1 - x_0)$

$(0\,m/s)^2 = (20\,m/s)^2 + 2(-9.8\,m/s^2)(x' - 0)$

$x' = 1.02\,m$

$V_f^2 = (0\,m/s)^2 + 2(-9.8\,m/s^2)(-10m - 1.02m)$

$\boxed{V_f = -14.70\,m/s}$

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Homework Continued

22.b) $\vec{v}_1 = \vec{v}_0 + a\,\Delta t$

$0\,m/s = 20\,m/s + (-9.8\,m/s^2)\Delta t = 2.04\,s$

$-14.70\,m/s = 0\,m/s + (-9.8\,m/s^2)\Delta t = 1.5\,s$

$2.04\,s + 1.5\,s = \boxed{3.54\,s}$

34. $v_x = 2t^2\,m/s$, $x_0 = 1\,m$, $t_0 = 0\,s$

a) $\Delta x = \int_{t_0}^{t_f} v_x(t)\,dt$

$x(t) = x_0 + \int_{t_0}^{t_f} 2t^2\,dt$

$x(t=1s) = 1 + \int_0^1 2t^2\,dt = 1 + 2\cdot\left[\frac{t^3}{3}\right]_0^1 = 1 + 2\cdot\frac{1}{3} = \boxed{\frac{5}{3}\,m}$

b) $v_x = 2(1)^2\,m/s = \boxed{2\,m/s}$

c) $a(t) = \frac{dv(t)}{dt} = 4t \Rightarrow 4(1) = \boxed{4\,m/s^2}$

37. Particle A) $\frac{-20-0}{7-5} = \boxed{-10\,m/s}$

Particle B) $\boxed{-20\,m/s}$

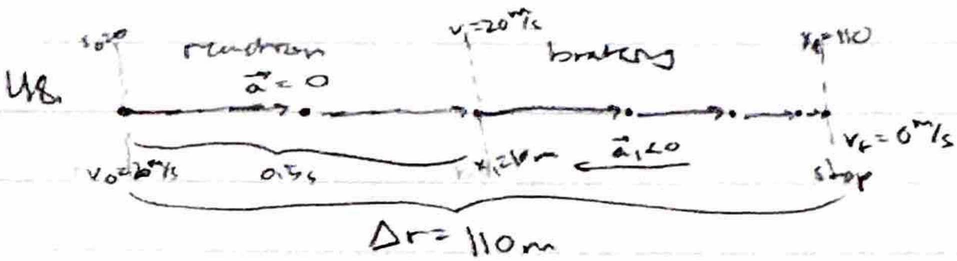
5/5 Particle C) $\Delta v = v_f - v_0 = \int_{t_0}^{t_f} a(t)\,dt$

or Area under curve

from 0 to 7s.

$A = 2\times30 + \frac{3\times30}{2} = 105 - 10\,m/s = \boxed{95\,m/s}$

48.

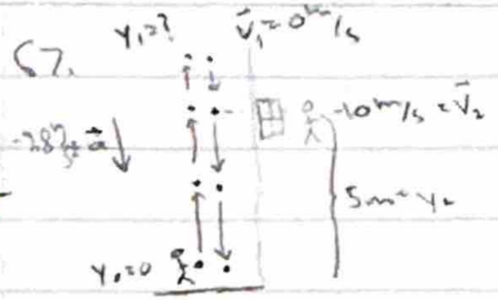


$x_1 = 0\,m + (20\,m/s)(0.5s - 0s) + 0 = 10\,m$

$(0\,m/s)^2 = (20\,m/s)^2 + 2a(110-10) = \boxed{-2\,m/s^2}$

5/5

67.



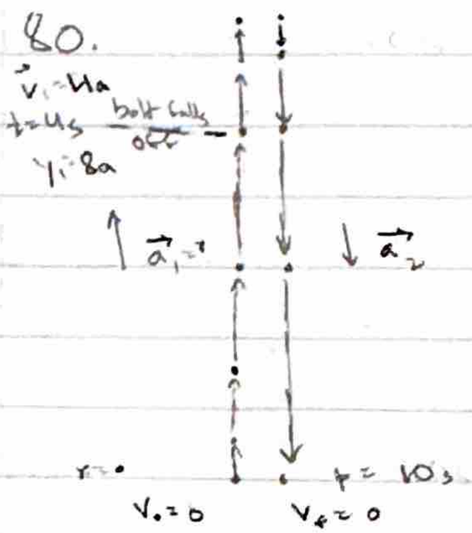
$v_2^2 = v_0^2 + 2a(y_2 - y_0)$

$(-10\,m/s)^2 = (v_0)^2 + 2(-9.8\,m/s^2)(5m - 0m)$

$100\,m^2/s^2 = v_0^2 - 98$

$\boxed{v_0 = 14.07\,m/s}$

80.



$v_1 = v_0 + a\Delta t$

$v_1 = 0\,m/s + a(4s) = 4a$

$y_1 = y_0 + v_{0y}\Delta t + \frac{1}{2}a(\Delta t)^2$

$y_1 = \frac{1}{2}a(4s)^2 = 8a$

$y_f = y_0 + v_{0y}\Delta t + \frac{1}{2}a(\Delta t)^2$

$0m = y_1 + v_1(6s) - \frac{1}{2}g(6s)^2$

$0m = 8a + 4a(6) + \frac{1}{2}(-9.8\,m/s^2)(6s)^2$

$\boxed{a = 5.5\,m/s^2}$

3.18, 3.19, 3.25 - Draw the decomposed vectors using dashed arrows

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3.33, 3.38 - Draw a simple picture including the vector of interest
Draw the decomposed vector using dashed arrows
Draw the coordinate system

Ch.2 EP# 27, 51, 62, 65 + Ch.3 EP# 3, 17, 18, 19, 25, 33, 38

Horizontal: $x_f = x_0 + v_{0x}\Delta t$

$v_{fx} = v_{0x}$

Vertical: $y_f = y_0 + v_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$

$v_{fy} = v_{0y} - g\Delta t$

$v_{fy}^2 = v_{0y}^2 - 2g(\Delta y)$

Ch.2

27. $\sin\theta\, g = a$

$a = \sin(10°)(-9.8 m/s) = -1.7$

$v_f^2 = v_0^2 + 2a(x_f - x_0)$

$0^2 = 30^2 + 2(-1.7)(x_f - 0) =$ $x_f = \boxed{264.71 m}$

51. $v_{fx}^2 = v_{0x}^2 + 2a(x_f - x_0)$

$0^2 = v_1^2 + 2(-1)(1-0) = \sqrt{2}\, m/s$

$\sqrt{2}^2 = v_2^2 + 2(-6)(2-0) = \sqrt{26}\, m/s$

$\sqrt{26}^2 = v_3^2 + 2(-1)(3-0) = \sqrt{32}\, m/s = \boxed{5.66 m/s}$

8/10

62. $x_f = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$. $\frac{1 km}{1 h} \times \frac{1000 m}{1 km} \times \frac{1 h}{60 min} \times \frac{1 min}{60 sec} = \frac{1000 m}{3600 sec}$

motion diagram -2

$0m + 18 km/h\,\Delta t + \frac{1}{2}(-0.2 m/s^2)(\Delta t)^2 = 200m - 6 km/h\,\Delta t + \frac{1}{2}(-0.4 m/s^2)(\Delta t)^2$

$0m + 5 m/s\,\Delta t + \frac{1}{2}(-0.2 m/s^2)(\Delta t)^2 = 200m - 1.67 m/s\,\Delta t + \frac{1}{2}(-0.4 m/s^2)(\Delta t)^2$

$5\Delta t - 0.1(\Delta t)^2 = 200 - 1.67\Delta t - 0.2(\Delta t)^2$

$-200 + 6.67\Delta t + 0.1(\Delta t)^2 = 0$

Back Page →

62. (continued) $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\frac{-6.67 \pm \sqrt{(-6.67)^2 - 4(0.1)(-200)}}{2(0.1)} = \frac{-6.67 \pm 11.16}{0.2} = 22.45$$

$\Delta t = 22.45\,s$

$x_f = 0\,m + 5\,m/s(22.45\,s) + \frac{1}{2}(-0.2\,m/s^2)(22.45\,s)^2$

$x_f = \boxed{61.85\,m}$

65. $v_{fx}^2 = v_{ox}^2 + 2a(x_f - x_0)$

$4.0^2\,m/s = 5.0^2\,m/s + 2a(3.0\,m - 0\,m)$

$16\,m/s = 25\,m/s + a(6\,m)$

$a = -1.5\,m/s^2$

$0^2\,m/s = 5.0^2\,m/s + 2(-1.5\,m/s^2)(x_f - 0)$

$0\,m/s = 25\,m/s + -3\,m/s^2\,(x_f)$

$x_f = 8.33\,m$

$8.5\,m - 8.33 = \boxed{0.17\,m \text{ or } 17\,cm}$

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Homework continued

Ch.3

3. a) $\vec{E} = \vec{E}_x + \vec{E}_y$

$\vec{E} = \vec{E}\sin\theta - \vec{E}\cos\theta$

$\boxed{\vec{E}_x = \vec{E}\sin\theta \text{ and } \vec{E}_y = -\vec{E}\cos\theta}$

b) $\vec{E} = \vec{E}_x + \vec{E}_y$

$\vec{E} = \vec{E}\cos\phi - \vec{E}\sin\phi$

$\boxed{\vec{E}_x = \vec{E}\cos\phi \text{ and } \vec{E}_y = \vec{E}\sin\phi}$

17. a) $\vec{E} = \sqrt{2\hat{i}^2 + 3\hat{j}^2} = \boxed{3.6}$

$\vec{F} = \sqrt{2\hat{i}^2 + (-2\hat{j})^2} = \boxed{2.8}$

6/6

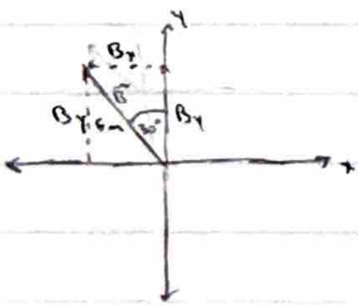
b) $\vec{E} + \vec{F} = 2\hat{i} + 3\hat{j} + 2\hat{i} - 2\hat{j} = 4\hat{i} + 1\hat{j}$

$\vec{E} + \vec{F} = \sqrt{4\hat{i}^2 + 1\hat{j}^2} = \boxed{4.1}$

c) $-\vec{E} - 2\vec{F} = -2\hat{i} - 3\hat{j} - 4\hat{i} + 4\hat{j} = -6\hat{i} + 1\hat{j}$

$-\vec{E} - 2\vec{F} = \sqrt{(-6\hat{i})^2 + 1\hat{j}^2} = \boxed{6.1}$

18. a)



$B_x = -5m\sin 30° = \boxed{-2.5m}$

$B_y = 5m\cos 30° = \boxed{4.3m}$

18. b)

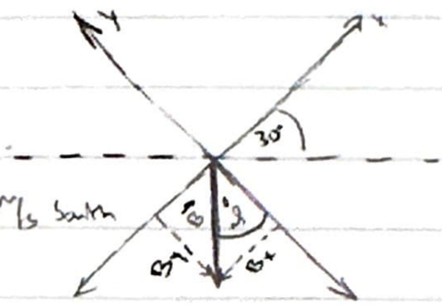
$B_x = \boxed{0\,m}$

$B_y = \boxed{5\,m}$

19.

3/4

$\vec{B} = 100\,m/s$ South



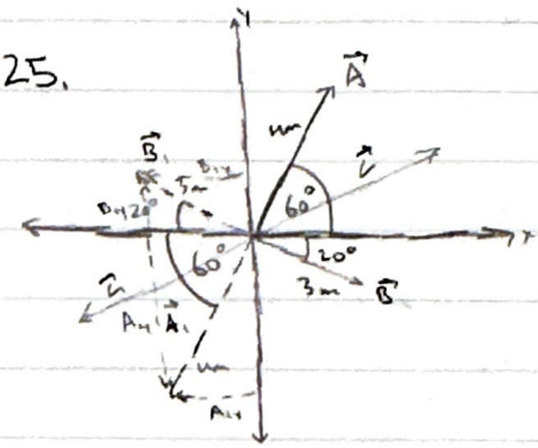
$\vec{v}\sin 30$

$B_x = -100\,m/s\,\sin 30° = \boxed{-50\,m/s}$

$B_y = -100\,m/s\,\cos 30° = \boxed{-86.6\,m/s}$

$\vec{v}\cos 30$

25.

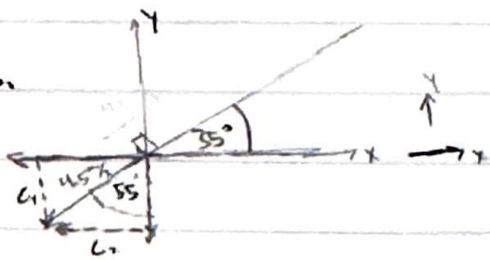


$\vec{A}_1 = -4\cos 60° - 4\sin 60° = -2\hat{i} - 3.5\hat{j}$

$\vec{B}_1 = 3\sin 20° - 3\cos 20° = -1.0\hat{j} + 2.8\hat{i}$

$\boxed{\vec{C}_1 = 0.8\hat{i} - 4.5\hat{j}}$

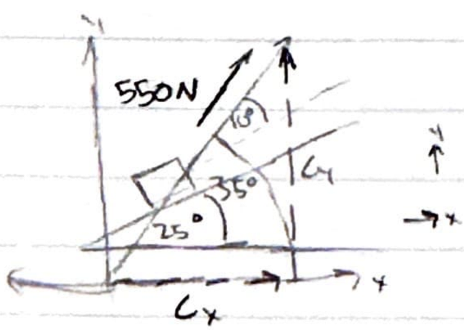
33.



$C_x = -4.5\sin 60° = \boxed{-3.7\,m/s}$

$C_y = -4.5\cos 60° = \boxed{-2.6\,m/s}$

38.



$C_x = 550\cos 35 = \boxed{450.5N}$

$C_y = 550\sin 35 = \boxed{315.5N}$

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Underline = Draw a picture/diagram/graph including the velocity vector and its decomposed vectors

~~18~~ 20
$\frac{}{20}$

Ch.4 # E.P. 6, 10, 13, 15, 27, 33, 36, 42, 50, 51, 56, 57, 72

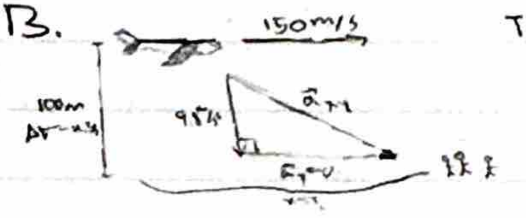
6. a) $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{30\,cm/s}{16\,cm/s}\right) = \boxed{62^\circ \text{ above the x-axis}}$

b) $x_1 = x_0 + \int_0^{5s} v_x\,dt = 0m +$ area under v_x-t curve $=$
$\frac{1}{2}(40cm/s)(5s) = 100cm$
$y_1 = y_0 + \int_0^{5s} v_y\,dt = 0m +$ area under v_y-t curve $=$
$(30\,cm/s)(5s) = 150cm$
$\Rightarrow r_1 = \sqrt{x_1^2 + y_1^2} = \sqrt{(100cm)^2 + (150cm)^2} = \boxed{180\,cm}$

10. a) $\vec{v} = (-3t\hat{i} + 2t^2\hat{j})\,m/s$ $\quad \vec{r}_0 = (3.0\hat{i} + 2.0\hat{j})\,m$
$\vec{r}(t) = \int \vec{v}\,dt + \vec{r}_0 = (-\frac{3}{2}t^2\hat{i} + \frac{2}{3}t^3\hat{j}) + (3.0\hat{i} + 2.0\hat{j})$
$\vec{r}(2) = (-\frac{3}{2}(2)^2\hat{i} + \frac{2}{3}(2)^3\hat{j}) + (3.0\hat{i} + 2.0\hat{j}) = \boxed{(-3.0\hat{i} + 7.3\hat{j})m}$

b) $\vec{a}$ = derivative of velocity $= -3\hat{i} + 4t\hat{j}$
$\vec{a}(2) = -3\hat{i} + 4(2)\hat{j} = \boxed{(-3\hat{i} + 8\hat{j})\,m/s^2}$

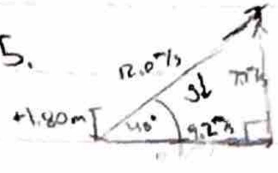
13.



Time of freefall $\Rightarrow y_f = y_0 + v_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$
$100m = 0m + 0m\Delta t - \frac{1}{2}(-9.8\,m/s^2)(\Delta t)^2$
$100m = 4.9\,m/s^2(\Delta t)^2 = \Delta t = 4.5s$

Distance package will travel in x direction $= (150\,m/s)(4.5s) = 675m$
$\Rightarrow$ In x direction, plane should drop before $\boxed{675m}$

15.



$\vec{v}_x = \cos(40)12.0\,m/s = 9.2\,m/s$
$\vec{v}_y = \sin(40)12\,m/s = 7.7\,m/s$

$y_f = y_0 + v_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$
$0m = 1.8m + 7.7\,m/s(\Delta t) - \frac{1}{2}g(\Delta t)^2 = \Delta t = 1.780s$

$y_f = y_0 + v_{0x}\Delta t \Rightarrow 0m + 9.2\,m/s(1.780s) = 16.376m \approx \boxed{16.4m}$

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27. a) $\frac{45 \text{ revolutions}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ rad}}{1 \text{ revolution}} = \boxed{4.7 \frac{\text{rad}}{s}}$

b) $T = \frac{2\pi \text{ rad}}{\omega} = \frac{2\pi \text{ rad}}{4.7 \frac{\text{rad}}{s}} = \boxed{1.3 s}$

33. a) $v = \frac{2\pi r}{T} = \frac{2\pi(1.5 \times 10^{11} m)}{\frac{365 \text{ days}}{1} \times \frac{24 hr}{1 d} \times \frac{60 min}{1 hr} \times \frac{60}{1 min}} = \boxed{3.0 \times 10^{4} m/s}$

b) $v = r\omega \Rightarrow \omega = \frac{v}{r} = \frac{3.0 \times 10^{4} m/s}{1.5 \times 10^{11} m/s} = \boxed{2.0 \times 10^{-7} \text{ rad}/s}$

c) $a_c = \frac{v^2}{r} = \frac{(3.0 \times 10^{4})^2}{(1.5 \times 10^{11})} = \boxed{6.0 \times 10^{-3} m/s^2}$

36. a) $\alpha = \frac{\Delta \omega}{\Delta t} = \frac{150-50}{1-0} = \boxed{100 \frac{\text{rads}}{s^2}}$

b) $\boxed{0 \frac{\text{rads}}{s^2}}$

c) $\alpha = \frac{\Delta \omega}{\Delta t} = \frac{250-150}{4-6} = \frac{100}{-2} = \boxed{-50 \frac{\text{rads}}{s^2}}$

42. a) $\omega_i = \frac{50 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} = 5.2 \text{ rad}/s$.

$\omega_f = \omega_0 + \alpha \Delta t \Rightarrow \omega_f = 5.2 \frac{\text{rads}}{s} + (0.5 \frac{\text{rads}}{s^2})(10 s) = 10.2 \frac{\text{rads}}{s}$.

$\frac{6}{6}$

$\frac{10.2 \text{ rads}}{s} \times \frac{60 s}{1 \text{ min}} \times \frac{1 \text{ rev}}{2\pi \text{ rads}} = \boxed{97.4 \text{ rpm}}$.

b) $\theta_f = \theta_i + \omega_0 \Delta t + \frac{1}{2}\alpha \Delta t^2 \Rightarrow \theta_f = 0 \text{ rads} + (5.2 \frac{\text{rads}}{s})(10 s) + \frac{1}{2}(0.5 \frac{\text{rad}}{s^2})(10 s)^2$

$= 77 \text{ rads} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} = \boxed{12.3 \text{ revolutions}}$.

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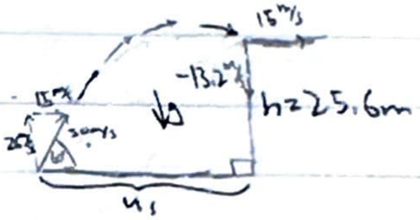
Ch. 4 Continued



50. a) $V_x = 30\cos 60° = 15\ m/s$

$V_y = 30\sin 60° = 26\ m/s$

$V_{fy} = V_{oy} - g\Delta t$

10/10

$V_{fy} = 26\ m/s - (9.8\ m/s^2)(4s) = -13.2\ m/s$

$y_f = y_o + V_{oy}\Delta t - \frac{1}{2}g(\Delta t)^2 = 0m + 26\ m/s(4s) - \frac{1}{2}(9.8\ m/s^2)(4s)^2$

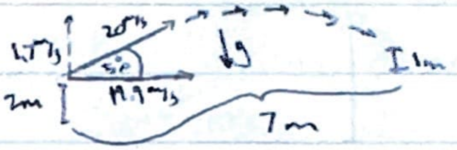
$y_f = \boxed{25.6m}$

b) $V_{fy}^2 = V_{oy}^2 - 2g(y_f - y_o)$

$(0\ m/s)^2 = (26\ m/s)^2 - 2(9.8\ m/s^2)(y_f - 0m) = y_f = \boxed{34.5m}$

c) $V_f = \sqrt{V_{fx}^2 + V_{fy}^2} = \sqrt{(15\ m/s)^2 + (-13.2\ m/s)^2} = \boxed{20.0\ m/s}$

51.



$V_{ox} = 20\cos 5° = 19.9\ m/s$

$V_{oy} = 20\sin 5° = 1.7\ m/s$

$x_f = x_o + V_{ox}\Delta t$

$7m = 0m + 19.9\ m/s\ \Delta t = 0.35s$

$y_f = y_o + V_{oy}\Delta t - \frac{1}{2}g(\Delta t)^2$

$y_f = 2m + 1.7\ m/s(0.35s) - \frac{1}{2}(9.8\ m/s^2)(0.35s)^2 = 2.0m$

Yes the ball will clear the net, it will be 0.1m higher than net.

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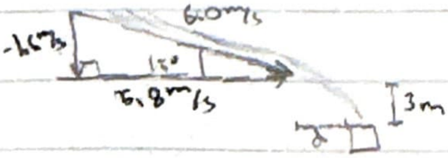
56.



$V_{x_0} = 6.0\,\text{m/s} \cos 15° = 5.8\,\text{m/s}$

$V_{y_0} = 6.0\,\text{m/s} \sin 15° = 1.6\,\text{m/s}$

$x_f = x_0 + V_{0x}\Delta t$

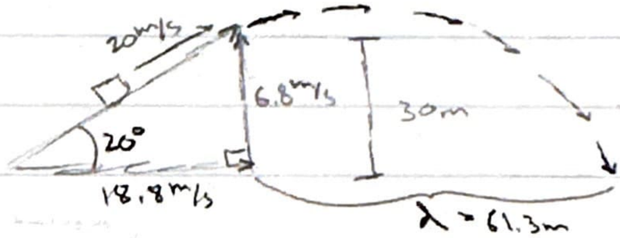
$y_f = y_0 + V_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$

$0 = 3 + 1.6\,\text{m/s}\,\Delta t - \frac{1}{2}(9.8\,\text{m/s}^2)(\Delta t)^2 = 3 - 1.6\Delta t - 4.9\Delta t^2$

$\Delta t = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{+1.6 \pm \sqrt{(-1.6)^2 - 4(-4.9)(3)}}{2(-4.9)} = 0.64\,\text{s}$

$x_f = 0 + 5.8\,\text{m/s}\,(0.64\,\text{s}) = \boxed{3.7\,\text{m}}$

57. a)



$V_{x_0} = 20\,\text{m/s} \cos 20° = 18.8\,\text{m/s}$

$V_{y_0} = 20\,\text{m/s} \sin 20° = 6.8\,\text{m/s}$

$y_f = y_0 + V_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$

$0 = 30 + 6.8\,\text{m/s}\,\Delta t - 4.9\,\text{m/s}^2(\Delta t)^2$

$\Delta t = \frac{-6.8 \pm \sqrt{(6.8)^2 - 4(-4.9)(30)}}{2(-4.9)} = 3.26\,\text{s}$

$x_f = x_0 + V_{0x}\Delta t = 0\,\text{m} + 18.8\,\text{m/s}(3.26\,\text{s})$

$x_f = \boxed{61.3\,\text{m}}$

b) $V_{fy} = V_{0y} - g\Delta t = 6.8\,\text{m/s} - 9.8\,\text{m/s}^2(3.26\,\text{s}) = -25.148\,\text{m/s}$

$V_f = \sqrt{V_{xf}^2 + V_{yf}^2} = \sqrt{(18.8)^2 + (-25.148)^2} = \boxed{31.4\,\text{m/s}}$

72. a) $w = (20 - \frac{1}{2}t^2)$, $0 = 20 - \frac{1}{2}t^2$, $t = \sqrt{40} = \boxed{6.3\,\text{s}}$

4/4

b) $\Delta\theta = \int_0^{6.3} 20 - \frac{1}{2}t^2\,dt = \left[20t - \frac{t^3}{6}\right]_0^{6.3} = \boxed{84\,\text{rads}}$

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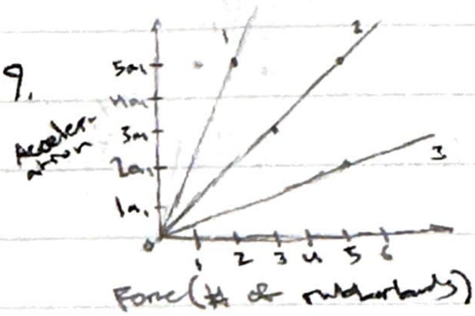
$\frac{20}{20}$ good job!

Ch. 5 E.P. # 9, 34, 42, 53, 56 + Ch. 6 E.P. # 2, 8, 9, 13, 18, 39, 40

Ch. 5 E.P. # 9, 34, 42, 53, 56

9.



$m_2 = 0.20\,kg$

4/4

$$\left.\begin{array}{l} 2F = m_1(5a_1) \\ 2F = m_2(2a_1) \end{array}\right\} 1 = \frac{5m_1}{2m_2} \Rightarrow m_1 = \frac{2}{5} m_2 = \frac{2}{5}(0.20\,kg) = \boxed{0.080\,kg}$$

$$\left.\begin{array}{l} 5F = m_3(2a_1) \\ 5F = m_2(5a_1) \end{array}\right\} 1 = \frac{2m_3}{5m_2} \Rightarrow m_3 = \frac{5}{2} m_2 = \frac{5}{2}(0.20\,kg) = \boxed{0.50\,kg}$$

34. a. Because $F \propto a$, if force is halved, then so is acceleration $\Rightarrow \frac{10\,m/s^2}{2} = \boxed{5\,m/s^2}$

b. Because $a \propto \frac{1}{m}$, if mass is halved, then acceleration is double $\Rightarrow 2(10\,m/s^2) = \boxed{20\,m/s^2}$

c. Because $F \propto a$ and $a \propto \frac{1}{m}$, then the halves cancel $\Rightarrow \boxed{10\,m/s^2}$

d. Because $F \propto a$ and $a \propto \frac{1}{m}$, the acceleration is halved twice $\Rightarrow \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(10\,m/s^2) = \boxed{2.5\,m/s^2}$

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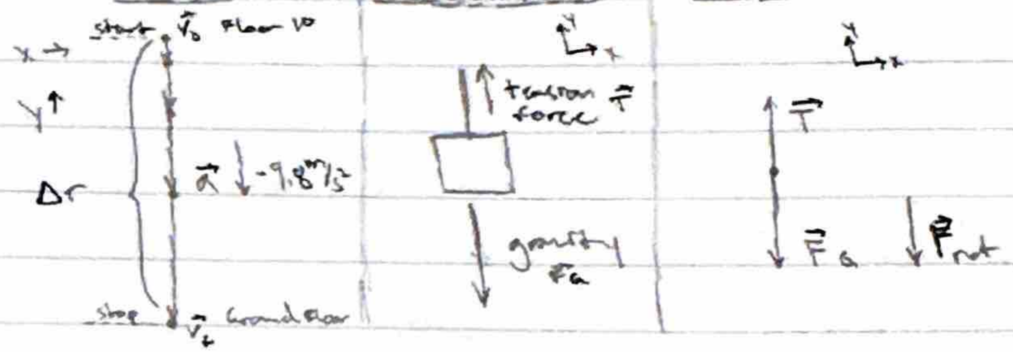
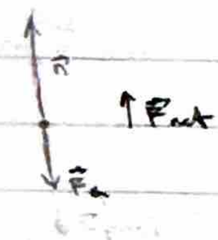
42. Motion | Force Identification | FBD



53. a) FBD



b) Because the leaf hopper has an upward acceleration of $4\,m/s^2$, it must have a positive vertical net force, where the only two forces are the normal force and gravity. therefore normal force > gravity.

56. a) There is no acceleration in the horizontal direction, which means no force. There is gravity and normal force in the vertical direction, but the net force is equal to zero

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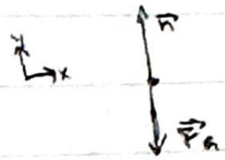
Homework continued

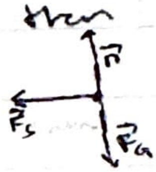
56. b)  $\vec{F}_{net}=0$, no net force

c) If the car is slowed down, then this does not affect me because the bench is frictionless.

d) If the car slows down, then so will the bench, but I will not because there is no friction. I will be going the same speed I was before the car slowed down.

e) Just because the car stops, does not mean I will too. Nothing pushes me forward, it just looks like that relative to the car.

f) If I stick (because of friction) to the slowing bench, then I will slow too.



$\vec{F}_{net}=\vec{F}_s$

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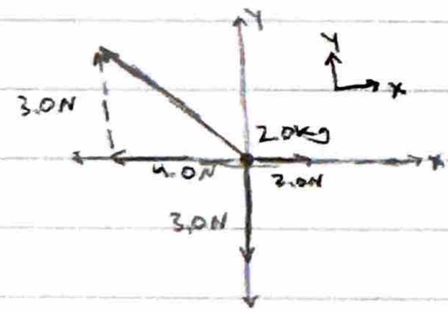
Ch. 6 EP # 2, 8, 9, 13, 18, 39, 40

2. 

$T_1 = 100 \sin 30° = \boxed{50N}$

$T_2 = 100 \cos 30° = \boxed{86.6N}$

8. 

$-3.0N + 3.0N = 0N$

$-4.0N + 2.0N = -2.0N$

$F = ma$

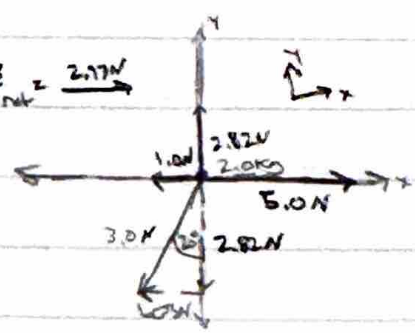
$-2.0N = (2.0kg)a_x$

$\vec{F}_{net} = \xleftarrow{2.0N}$

$\boxed{\vec{a}_x = -1.0 m/s^2 \quad \vec{a}_y = 0 m/s^2}$

6/6

9. $\vec{F}_{net} = \xrightarrow{2.97N}$



$3 \sin 20° = 1.03N$

$3 \cos 20° = 2.82N$

$\vec{F}_x = 5N - 1N - 1.03N = 2.97N$

$\vec{F}_y = 2.82N - 2.82N = 0N$

$F = ma \Rightarrow 2.97N = (2.0kg)\vec{a}_x =$ $\boxed{\vec{a}_x = 1.485 m/s^2 \quad \vec{a}_y = 0 m/s^2}$

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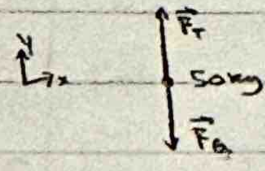
Homework Continued

13. a)  $\vec{F}_T = ma$

$\vec{F}_T = 50kg(9.8\,m/s^2) = \boxed{490N}$

b) Because the box is moving at a constant speed then $a = 0\,m/s^2 \Rightarrow$ forces don't change,

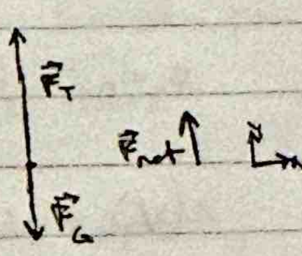
$\boxed{\vec{F}_T = 490N}$

c) $F = ma \Rightarrow (5.0\,m/s^2)(50kg) = 250N$

$250N + 490N = \boxed{\vec{F}_T = 740N}$

d) $F = ma \Rightarrow (-5.0\,m/s^2)(50kg) = 250N$

$490N - 250N = \boxed{\vec{F}_T = 240N}$

18. a) $w = mg\left(1 + \frac{a_y}{g}\right)$ 

$w = (60kg)(9.8\,m/s^2) = \boxed{590N}$

b) $a_y = \frac{\Delta v}{\Delta t} = \frac{10\,m/s}{4s} = 2.5\,m/s^2$

$w = (60kg)(9.8\,m/s^2)\left(1 + \frac{2.5\,m/s^2}{9.8\,m/s^2}\right) = \boxed{738N}$

c) Cruising speed = no new force

$\Rightarrow \boxed{590N}$

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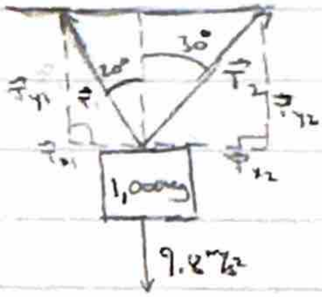
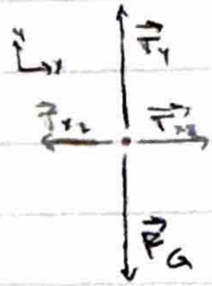
$\frac{10}{10}$

39.





$F_{net} = 0$

$$\vec{F}_{net} = \vec{T}_1 + \vec{T}_2 + \vec{F}_G = 0N$$
$$\vec{F}_{net_x} = \vec{T}_{1x} + \vec{T}_{2x} + \vec{F}_{Gx} = 0N$$
$$\vec{F}_{net_y} = \vec{T}_{1y} + \vec{T}_{2y} + \vec{F}_{Gy} = 0N$$

$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0N$$
$$T_1 \cos\theta + T_2 \cos\theta_2 - F_G = 0N$$

$$-T_1 \sin 20° + T_2 \sin 30° = 0N \quad \Big| \quad T_1 \cos 20° + T_2 \cos 30° = 9800N$$

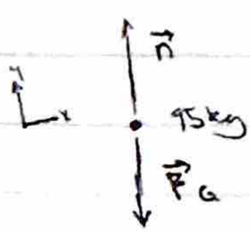
$\boxed{T_1 = 6397N}$ $\quad$ $\boxed{T_2 = 4376N}$

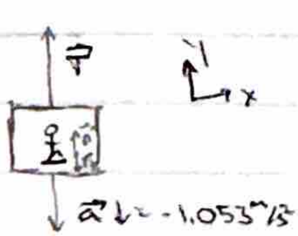
40. $F = ma$ $\quad$ original weight $= (95kg)(9.8 m/s^2) = 930N$

$$F_y = n - mg = ma_y \Rightarrow a_y = \frac{n-mg}{m} = \frac{830N - 930N}{95kg} = -1.053 m/s^2$$

$$(v_y)_f = (v_y)_i + a_y \Delta t = (-1.053 m/s^2)(3.03s) = \boxed{-3.2 m/s}$$

Because velocity doesn't change in the last three seconds





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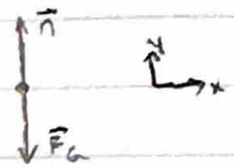
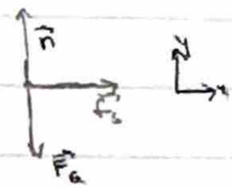
Ch. 6 # E.P. 26, 27, 50, 51, 54, 56, 58 + Ch. 7 # E.P. 9, 14, 17, 23, 24, 25

Ch. 6 # E.P. 26, 27, 50, 51, 54, 56, 58

26. a)

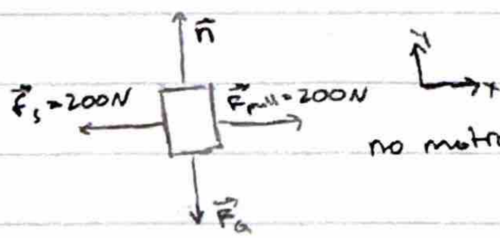


b)



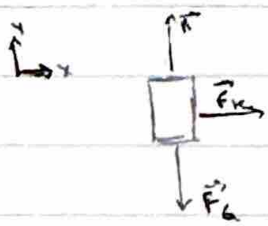
c) $f_{s\,max} = \mu_s n \Rightarrow a_{max} = \frac{f_{s\,max}}{m} = \frac{\mu_s n}{m} = \frac{(0.5)(9.8 m/s^2)(10kg)}{10kg} = \boxed{4.9 m/s^2}$

27.



no motion $\Rightarrow$ $\vec{F}_s = \boxed{200N}$

50.



$\vec{F}_G = (10kg)(9.8 m/s^2) = 98N$

$\vec{n} = 98N$

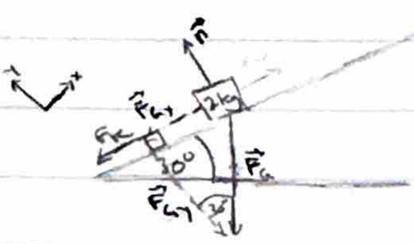
$\mu_s = 0.5 \quad \mu_k = 0.3$

$\vec{f}_k = \mu_k mg = (0.3)(10kg)(9.8 m/s^2) = 29.4N$

$a = \frac{f_k}{m} = 2.94 m/s^2 \Rightarrow v_1^2 = v_0^2 + 2a\Delta x$

$\Rightarrow (2 m/s)^2 = (0 m/s)^2 + 2(2.94 m/s^2)\Delta x = \Delta x = \boxed{0.68m}$

51. a)



$\vec{F}_{Gy} = -19.6\cos 30 = -8.49m = -16.18N$

$\vec{F}_{Gx} = -19.6\sin 30 = -4.9m = -9.8N$

$\vec{f}_k = \mu_k \vec{n} \Rightarrow$

$a_x = \frac{\vec{F}_{net}}{m} = \frac{\vec{F}_G - f_k}{m} = \frac{-mg\sin\theta + \mu_k mg\cos\theta}{m} = -g(\sin\theta + \mu_k\cos\theta) = -9.8(\sin 30 + 0.2\cos 30) = -6.6 m/s^2$

$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow (0)^2 = (10^2) + 2(-6.6)\Delta x = \Delta x = 7.6m \Rightarrow h = \Delta x\sin\theta = 7.6\sin 30 = \boxed{3.8m}$

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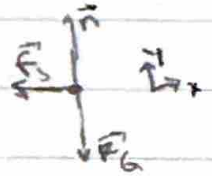
51. b) $a_x = \frac{(F_{net})_x}{m} = \frac{-F_G\sin\theta + f_k}{m} = \frac{-mg\sin\theta + f_k}{m}$

$a_x = -g(\sin\theta - \mu_k\cos\theta) = -3.2\,m/s^2$

$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow v_1 = \sqrt{(0\,m/s)^2 + 2(-3.2\,m/s^2)(-7.6\,m)} = \boxed{-7.0\,m/s}$

54.



$\Sigma F_y = n - mg = 0 \Rightarrow n = mg$

$\Sigma F_x = -f_s = m a_x$

$\Delta x = d_{min}$

$v_1 = 0\,m/s$

$-(f_s)_{max} = -\mu_s n = -\mu_s mg = m a_x$

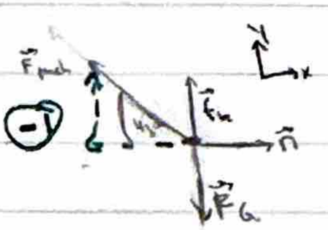
$a_x = -\mu_s g$

$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow 0 = v_0^2 + 2(-\mu_s g)d_{min} \Rightarrow -v_0^2 = 2(-\mu_s g)d_{min}$

$\Rightarrow \boxed{\frac{v_0^2}{(\mu_s g)} = d_{min}}$

$\frac{9}{10}$

56 -1



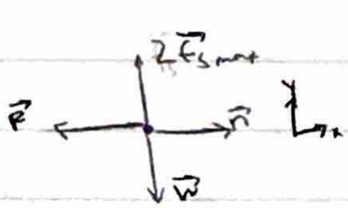
$\vec{a} = 0 \Rightarrow F_{net_x} = 0N = n - F_{push}\cos 45°$

$F_{net_y} = 0N = f_k + F_{push}\sin 45° - F_G = f_k + F_{push}\sin 45° - mg$

$f_k = \mu_k F_{push}\cos 45°$

$\mu_k F_{push}\cos 45° + F_{push}\sin 45° - mg = 0N \Rightarrow F_{push} = \frac{mg}{\mu_k\cos 45° + \sin 45°} = \frac{(2.0\,kg)(9.8\,m/s^2)}{0.2\cos 45° + \sin 45°} = \boxed{23N}$

58.



$\vec{f}_{s\,max} = \mu_s n = (0.8)(6.0N) = 4.8N$

$w = f_{s\,max} + f_{s\,max}$

$w = 2f_{s\,max} = 2(4.8N) = \boxed{9.6N}$

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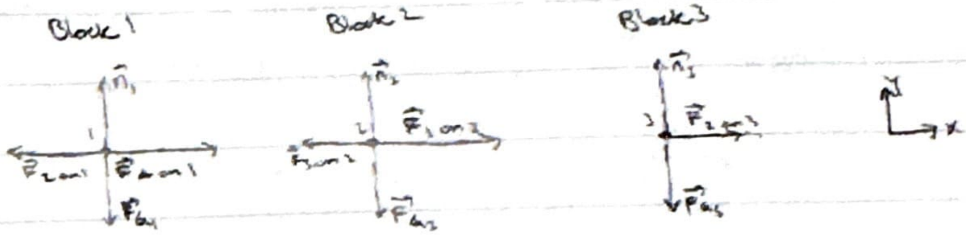
Homework continued

Ch. 7 #EP. 9, 14, 17, 23, 24, 25

9. a)

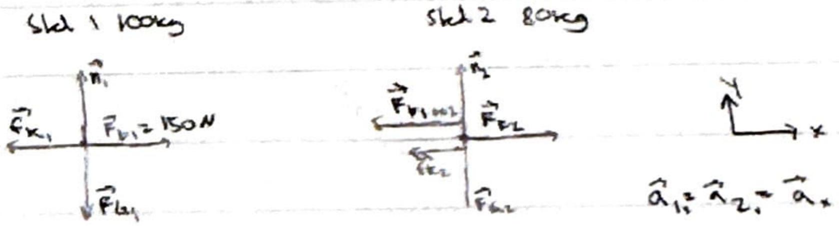


$\vec{F}_{A\,on\,1} = (m_1 + m_2 + m_3)a \Rightarrow 12N = (1kg + 2kg + 3kg)a = a = 2\,m/s^2$

$\vec{F}_{2\,on\,3} = m_3 a = (3kg)(2\,m/s^2) = \boxed{6N}$

b) $12N - \vec{F}_{2\,on\,1} = (1kg)(2\,m/s^2) = \boxed{10N}$

14



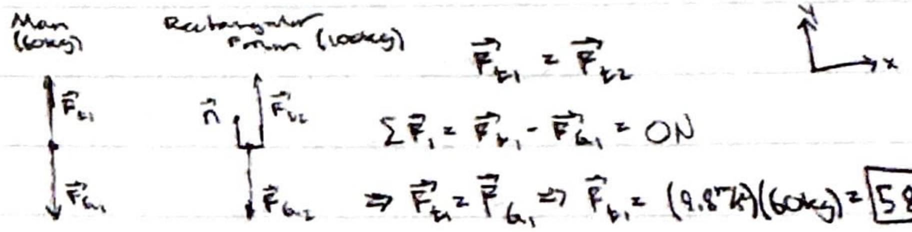
$\vec{F} = ma$ $\qquad \vec{f}_K = \mu_K n$

$\Sigma \vec{F}_{x_1} = \vec{F}_{t_1} - \vec{f}_{K_1} = m_1 a_x \Rightarrow a_x = \frac{150N - (100)(9.8)(0.1)}{100} = 0.52\,m/s^2$

$\Sigma \vec{F}_{x_2} = \vec{F}_{t_2} - \vec{F}_{t\,1\,on\,2} - \vec{f}_{K_2} = m_2 a_2 \Rightarrow \vec{F}_{t_2} - 150N - (80kg)(9.8\,m/s^2)(0.1) = (80kg)(0.52\,m/s^2)$

$\Rightarrow \vec{F}_{t_2} = \boxed{270N}$

17.



$\vec{F}_{t_1} = \vec{F}_{t_2}$

$\Sigma \vec{F}_1 = \vec{F}_{t_1} - \vec{F}_{G_1} = 0N$

$\Rightarrow \vec{F}_{t_1} = \vec{F}_{G_1} \Rightarrow \vec{F}_{t_1} = (9.8\,m/s^2)(60kg) = \boxed{588N}$

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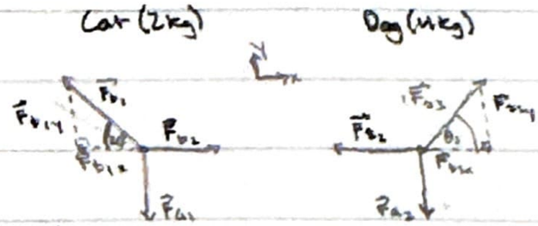
23.



$\vec{F}_{t2} = \vec{F}_{t1x} = \vec{F}_{t3x} = 52.2N$

$\vec{F}_{G1} = (9.8)(2) = 19.6N = \vec{F}_{t1y}$

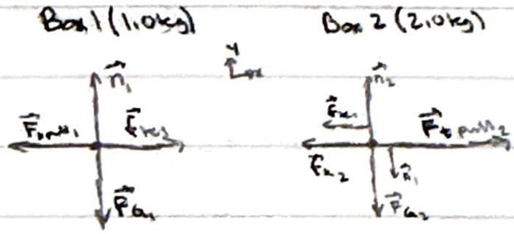
$\vec{F}_{G2} = (9.8)(4) = 39.2N = \vec{F}_{t3y}$

$\tan 20° = \frac{F_{t1y}}{F_{t1x}} \Rightarrow F_{t1x}\tan 20° = F_{t1y} \Rightarrow F_{t1x} = \frac{F_{t1y}}{\tan 20} = \frac{19.6N}{\tan 20} = 52.2N$

$\theta_3 = \tan^{-1}\left(\frac{F_{t3y}}{F_{t3x}}\right) = \tan^{-1}\left(\frac{39.2N}{52.2N}\right) = \boxed{36.9°}$

$\vec{F}_{t3} = \sqrt{(39.2)^2 + (52.2)^2} = \boxed{65.28N}$

24. a)



$\vec{F}_{tpull_1} = \vec{f}_{k_1} = \vec{n}_1 \mu_k = (1kg)(9.8 m/s^2)(0.4) = 3.92$

$\vec{F}_{tpull_1} = \boxed{3.92N}$

- Everything is alright

PPAP?

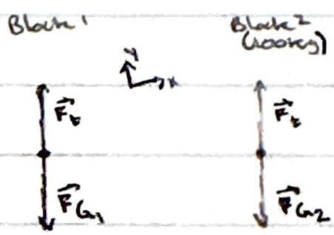
b) $\sum \vec{F}_{2x} = m\vec{a} = \vec{F}_{tpull_2} - \vec{f}_{k_1} - \vec{f}_{k_2} \Rightarrow m\vec{a} = 20N - 3.92N - \vec{n}_2 \mu_k$

$= m\vec{a} = 20N - 3.92N - (29.4N)(0.40) = m\vec{a} = 4.32N$

$\Rightarrow \vec{a} = \frac{4.32N}{m} = \frac{4.32N}{2kg} = \vec{a} = \boxed{2.16 m/s^2}$

25.

10/10



$\sum \vec{F}_{1y} = \vec{F}_t - \vec{F}_{G1}$

$\sum \vec{F}_{2y} = \vec{F}_t - \vec{F}_{G2}$

$y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2$

$-1m = 0m + 0 m/s (6s) + \frac{1}{2}(a_y)(6s)^2$

$\vec{a}_y = \vec{a}_2 = -0.0556 m/s^2$

$-Mg + mg = Ma_m + ma_m$

$m = M\left(\frac{g + a_m}{g - a_m}\right) = (100kg)\left(\frac{9.8 - 0.0556}{9.8 + 0.0556}\right) = \boxed{99 kg}$

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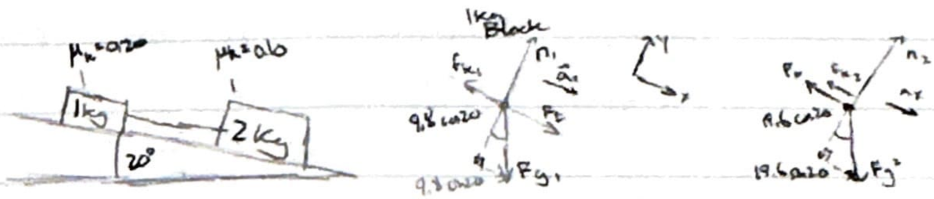
$\frac{19.75}{20}$

HW7 - Ch.7 # EP 34, 38, 39, 40, 42, 45, 49 + Ch.8 # EP 12, 13, 19, 25, 40, 46, 56

Ch.7 # EP 34, 38, 39, 40, 42, 45, 49

34.



$\Sigma F_{1x} = F_t + 9.8\sin 20 - f_{k1}$

$\Sigma F_{1x} = F_t + 9.8\sin 20 - \mu_k n_1$

$\Sigma F_{1x} = F_t + 9.8\sin 20 - (0.20)(9.8\cos 20)$

$m_1 a_x = F_t + 9.8\sin 20 - (0.20)(9.8\cos 20)$

$\Sigma F_{2x} = 19.6\sin 20 - F_t - f_{k2}$

$\Sigma F_{2x} = 19.6\sin 20 - F_t - \mu_2 n_2$

$\Sigma F_{2x} = 19.6\sin 20 - F_t - (0.1)(19.6\cos 20)$

$m_2 a_x = 19.6\sin 20 - F_t - (0.1)(19.6\cos 20)$

$$\frac{F_t + 9.8\sin 20 - (0.20)9.8\cos 20}{m_1} = \frac{19.6\sin 20 - F_t - (0.1)(19.6\cos 20)}{m_2}$$

$F_t + \cancel{3.35} - 1.84 = \cancel{3.35} - \frac{1}{2}F_t - 0.92$

$1.5F_t - 1.84 = -0.92$

$1.5F_t = 0.92$

$F_t = \boxed{0.61\text{N}}$

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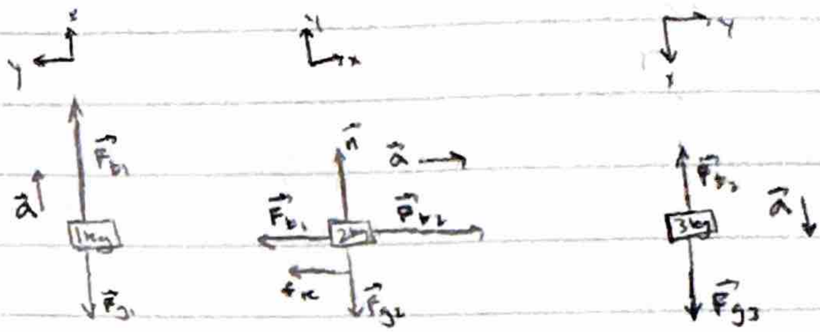
38.



$\Sigma F_{2x} = m_2 a = F_{t2} - F_{t1} - f_k$

$\Sigma F_{1x} = m_1 a = F_{t1} - m_1 g \Rightarrow F_{t1} = m_1 a + m_1 g$

$\Sigma F_{3x} = m_3 a = m_3 g - F_{t2} \Rightarrow F_{t2} = m_3 g - m_3 a$

$\Sigma F_{2x} = m_2 a = (m_3 g - m_3 a) - (m_1 a + m_1 g) - (\mu_k m_2 g)$

$\Sigma F_{2x} = m_2 a = m_3 g - m_3 a - m_1 a - m_1 g - \mu_k m_2 g$

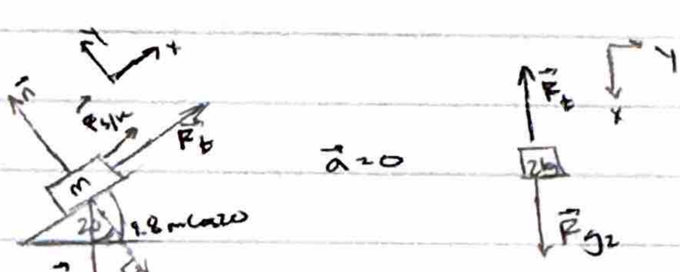
$m_2 a + m_3 a + m_1 a = m_3 g - m_1 g - \mu_k m_2 g$

$a(m_1 + m_2 + m_3) = m_3 g - m_1 g - \mu_k m_2 g$

$a = \frac{m_3 g - m_1 g - \mu_k m_2 g}{m_1 + m_2 + m_3}$

$a = \boxed{2.28\,m/s^2}$

39.



a) $\vec{F}_{g2} = m_2 g = \vec{F}_t = 19.6\,N$ $\quad$ $\vec{F}_s = \mu_s 9.8 m \cos 20$

$\Sigma \vec{F}_x = -9.8 m \sin 20 + \vec{F}_t + \vec{F}_s = 0$ $\quad$ $\vec{F}_s = (0.4)(9.8 m \cos 20)$

$9.8 m \sin 20 = -19.6 + (0.8)(9.8 m \cos 20)$

$9.8\, m \sin 20 = -19.6 + 7.84\, m \cos 20 \Rightarrow m(9.8 \sin 20 + 7.84 \cos 20) = 19.6$

$\Rightarrow m = \frac{19.6}{(9.8 \sin 20 + 7.84 \cos 20)} = \boxed{1.83\,kg}$

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HW7 - Continued

39. b) $\Sigma \vec{F} = ma = \vec{F}_t - \vec{F}_k - \vec{F}_{gx}$

$\Sigma \vec{F} = ma = \vec{F}_t - \mu_k 9.8m\cos 20 - 9.8m\sin 20$

$\Sigma \vec{F} = Ma = Mg - \vec{F}_t$

$\vec{F}_t = Mg - Ma$

$\Sigma \vec{F} = ma = Mg - Ma - \mu_k 9.8m\cos 20 - 9.8m\sin 20$

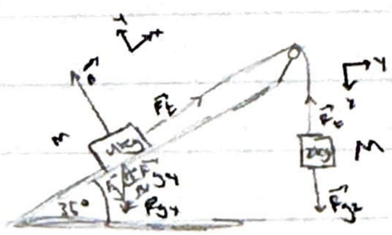
$\Sigma \vec{F} = ma + Ma = Mg - \mu_k 9.8m\cos 20 - 9.8m\sin 20$

$a(m+M) = Mg - \mu_k 9.8m\cos 20 - 9.8m\sin 20$

$a = \dfrac{Mg - \mu_k 9.8m\cos 20 - 9.8m\sin 20}{m+M}$

$a = \dfrac{(9.8)(2 - 0.5(1.83\cos 20) - 1.83\sin 20)}{1.83 + 2} = \boxed{1.32\ m/s^2}$

40.



a) $\vec{F}_t = \vec{F}_{gz} = Mg = 9.8(2) = \boxed{19.6N}$

b) $\vec{F}_{gx} = (4)(9.8)\sin(35) = 22.5N \Rightarrow 22.5N > 19.6N \Rightarrow$ $\boxed{\text{Down the Ramp}}$

c) $\Sigma \vec{F}_x = ma_x = \vec{F}_t - mg\sin 35 = Mg - Ma - mg\sin 35 = ma$

$\Sigma \vec{F}_y = Ma_y = Mg - \vec{F}_t \Rightarrow \vec{F}_t = Mg - Ma$

$a = \dfrac{Mg - mg\sin 35}{M+m} = \dfrac{g(M - m\sin 35)}{M+m}$

$a = \boxed{-0.48\ m/s^2}$

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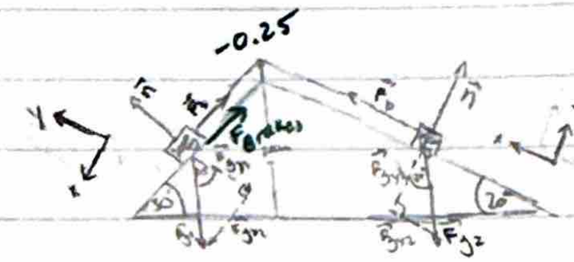
42. -0.25



9.75 / 10

a) $\vec{F}_{Brakes} = \vec{F}_{gx1} - \vec{F}_t$ $\qquad \vec{F}_{gx_1} = (9.8)(2000)\sin 30 = 9800N$

$\Sigma \vec{F}_1 = \vec{F}_{gx_1} - \vec{F}_t - \vec{F}_{Brakes} = ma = 0 \Rightarrow \vec{F}_{Brakes} = \vec{F}_{gx1} - \vec{F}_t$

$\Sigma \vec{F}_2 = \vec{F}_t - \vec{F}_{gx2} = ma = 0 \Rightarrow \vec{F}_t = \vec{F}_{gx2} = 6033.24N$

$\vec{F}_{Brakes} = 9800N - 6033.24N = \boxed{3767N}$

b) $\Sigma \vec{F} = m_{total}\, a \Rightarrow \frac{3767N}{(2000+1800)} = 0.991 m/s^2$

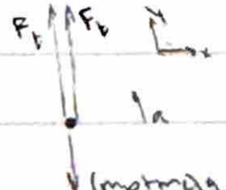
$x_f = x_0 + v_{0x}\Delta t + \frac{1}{2}a(\Delta t)^2 \qquad x_0 = 0m \quad x_f = \frac{200m}{\sin 35} = 400m$

$400m = 0m + 0 m/s(\Delta t) + \frac{1}{2}(0.991 m/s^2)(\Delta t)^2 \qquad V_{0x} = 0 m/s \quad a = 0.991 m/s^2$

$400m = 0.496 m/s^2 (\Delta t)^2 \Rightarrow \Delta t = 28.41s$

$V_{fx} = (28.41s)(0.991 m/s^2) = \boxed{28.16 m/s}$

45.

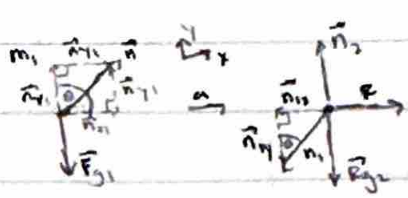


$\Sigma \vec{F} = m_{system}\, a \Rightarrow 2T - (m_p+m_c)g = (m_p+m_c)a$

$T = \frac{1}{2}(m_p+m_c)(a+g)$

$T = \frac{1}{2}(70+10)(0.2+9.8) = \boxed{400N}$

49.



$\Sigma \vec{F}_{x_1} = m_1 a_x = n_{x1} = n_1 \sin\theta$

$\Sigma \vec{F}_{x_2} = m_2 a_x = \vec{F} - n_{x1} = \vec{F} - n_1 \sin\theta$

$\Sigma \vec{F}_{y_1} = m_1 a_y = n_{1y} - m_1 g = n_1 \cos\theta - m_1 g = 0$

$n_1 = \frac{m_1 g}{\cos\theta} \Rightarrow \frac{m_1 g}{\cos\theta}\sin\theta = m_1 a \Rightarrow a = g\frac{\sin\theta}{\cos\theta} = g\tan\theta$

$F = m_2 a + n_2 \sin\theta = m_2 a + n_1 \sin\theta$

$\Rightarrow F = m_2(g\tan\theta) + \left(\frac{m_1 g}{\cos\theta}\right)\sin\theta = m_2 g\tan\theta + m_1 g\tan\theta \Rightarrow \boxed{F = (m_1+m_2)g\tan\theta}$

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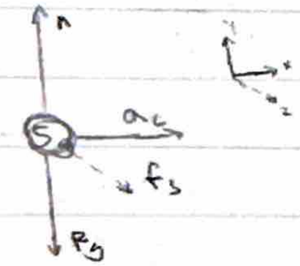
HW7 - Continued

Ch. 8 # BP 12, 13, 19, 25, 40, 46, 56

12.



$\Sigma \vec{F}_x = m a_c$

$(\vec{f}_s)_{max} = m r \omega^2_{max}$

$\mu_s n = m r \omega^2_{max}$

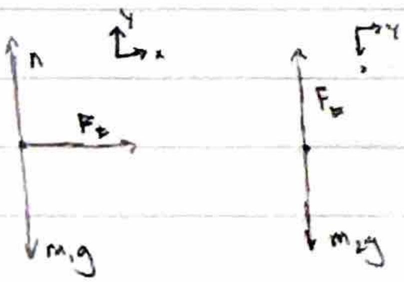
$\mu_s \cancel{m} g = \cancel{m} r \omega^2_{max}$

$\omega_{max} = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{0.8 \times 9.8}{0.15}} = 7.23 \text{ rad/s}$

$\frac{7.23 \text{ rad}}{1s} \times \frac{60s}{1 \text{ minute}} \times \frac{1 \text{ rev}}{2\pi \text{ rad}} =$ 69 rpm > 60 rpm

$\boxed{\text{The coin does not slip}}$

13.



$\Sigma \vec{F}_2 = m_2 g - \vec{F}_t = 0$

$m_2 g = \vec{F}_t$

$\Sigma \vec{F}_1 = m_1 a_c$

$\Sigma \vec{F}_1 = m_1 \frac{v^2}{r} = \vec{F}_t = m_2 g$

$m_2 g = m_1 \frac{v^2}{r} \Rightarrow \boxed{v = \sqrt{\frac{r m_2 g}{m_1}}}$

19.

↑a ↑+ 50% increase weight $n = 1.5 \vec{F}_g$

$\Sigma \vec{F} = m a_c$

$\Sigma \vec{F} = n - mg = m\left(\frac{v^2}{r}\right)$

$\Sigma \vec{F} = 1.5 mg - mg = m\left(\frac{v^2}{r}\right)$

$0.5 \cancel{m} g = \cancel{m}\left(\frac{v^2}{r}\right)$

$0.5 g = \frac{v^2}{r}$

$v = \sqrt{r \, 0.5 g} = \sqrt{(30)(0.5)(9.8)} = \boxed{12.1 \text{ m/s}}$

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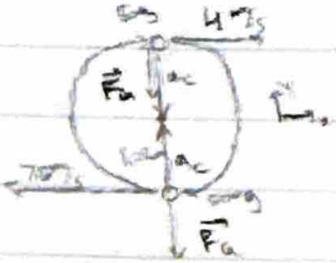
25.



a) $\vec{F}_G = mg = 0.5kg \cdot 9.8 m/s^2 = \boxed{4.9N}$

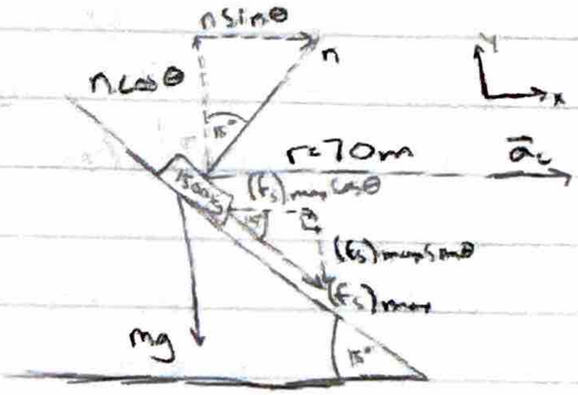
b) $\vec{F} = ma = m\frac{v^2}{r} = 0.5\frac{4^2}{1.02} = 7.84N$

$\vec{F}_t = 7.84N - \vec{F}_G = \boxed{2.94N}$

c) $\vec{F} = ma = m\frac{v^2}{r} = 0.5\frac{7.5^2}{1.02} = 27.57$

$\vec{F}_t = 27.57 + \vec{F}_G = \boxed{32.47N}$

40.



$\sum \vec{F}_r = ma_c$

$n\sin\theta + (f_s)_{max}\cos\theta = m\frac{v^2}{r}$

$n\sin\theta + \mu_s n\cos\theta = m\frac{v^2}{r}$

$\sum \vec{F}_y = ma_y$

$n\cos\theta - (f_s)_{max}\sin\theta - mg = 0$

$n\cos\theta - \mu_s n\sin\theta - mg = 0$

$\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$

$v = \sqrt{(70m)(9.8 m/s^2)\frac{\sin 15° + (1)\cos 15°}{\cos 15° - (1)\sin 15°}}$

$v = \boxed{34.5 m/s}$

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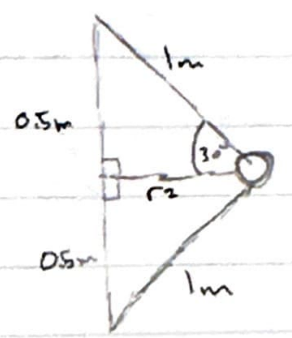
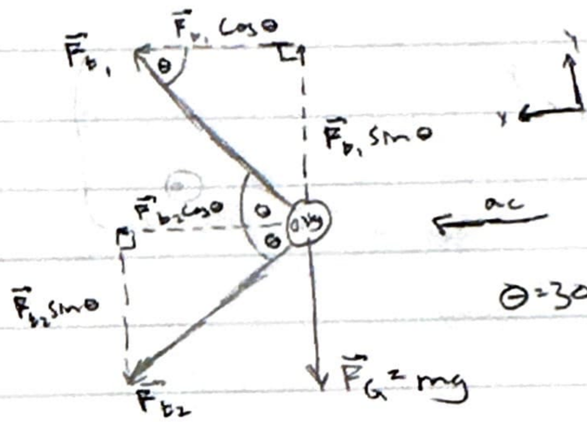
HW 7 - Continued

46. $\sin^{-1}\left(\frac{0.5}{1}\right) = 30°$





$\theta = 30°$

10/10

$\Sigma \vec{F}_x = m a_c$

$\Sigma \vec{F}_x = \vec{F}_{t_1}\cos\theta + \vec{F}_{t_2}\cos\theta = m\frac{v^2}{r}$

$(\vec{F}_{t_1} + \vec{F}_{t_2})\cos\theta = m\frac{v^2}{r}$

$\vec{F}_{t_1} + \vec{F}_{t_2} = \frac{mv^2}{r\cos\theta}$

$\Sigma \vec{F}_y = m a_y$

$\Sigma \vec{F}_y = \vec{F}_{t_1}\sin\theta - \vec{F}_{t_2}\sin\theta - \vec{F}_G = 0$

$(\vec{F}_{t_1} - \vec{F}_{t_2})\sin\theta = mg$

$\vec{F}_{t_1} - \vec{F}_{t_2} = \frac{mg}{\sin\theta}$

$2\vec{F}_{t_1} = \frac{mv^2}{r\cos\theta} + \frac{mg}{\sin\theta}$ $\qquad r = (1.0m)\cos\theta$

$\vec{F}_{t_1} = \frac{m}{2}\left(\frac{v^2}{r\cos\theta} + \frac{g}{\sin\theta}\right)$

$\vec{F}_{t_1} = \frac{0.3kg}{2}\left(\frac{(7.5\,m/s)^2}{((1m)\cos^2 30°} + \frac{(9.8\,m/s^2)}{(\sin 30°)}\right) = \boxed{14.2N = \vec{F}_{t_1}}$

$\vec{F}_{t_2} = \vec{F}_{t_1} - \frac{mg}{\sin\theta} = 14.2N - \frac{(0.3kg)(9.8\,m/s^2)}{\sin 30} = \boxed{8.32N = \vec{F}_{t_2}}$

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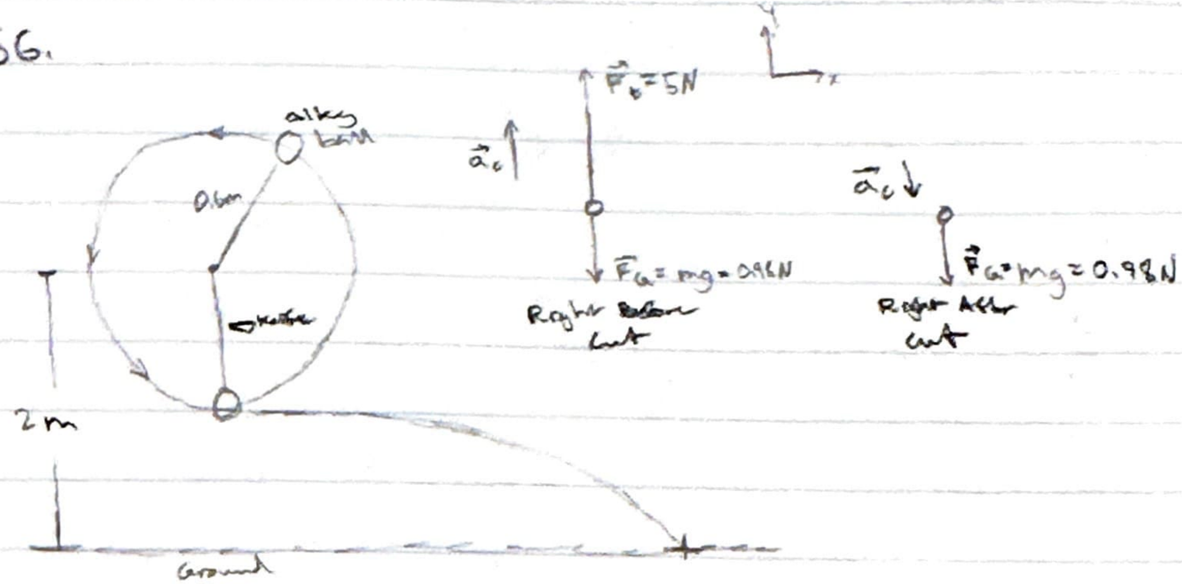
56.



$\Sigma \vec{F} = ma_c$

$\vec{F}_t - \vec{F}_G = m\frac{v^2}{r}$

$v = \sqrt{\frac{r(\vec{F}_t - \vec{F}_G)}{m}}$

$v = \sqrt{\frac{(0.6m)(5N - 0.98N)}{0.1kg}} = 4.91 m/s$

$y_f = y_0 + V_{0y}\Delta t - \frac{1}{2}g(\Delta t)^2$

$0 = 1.4m + (0 m/s)\Delta t - \frac{1}{2}(9.8 m/s^2)(\Delta t)^2$

$\Delta t = 0.534s$

$x_f = x_0 + V_{0x}\Delta t$

$x_f = 0m + (4.91 m/s)(0.534s)$

$x_f = \boxed{2.62m}$

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$\frac{20}{20}$

HW8 Ch. 9 # E.P. 12, 13, 15, 17, 18, 20, 21, 27, 33, 37, 40, 45, 56

12. a) $\vec{A} = 3\hat{i} + 4\hat{j}$ $\quad \vec{B} = 2\hat{i} - 6\hat{j}$

$\vec{A} \cdot \vec{B} = (3)(2) + (4)(-6) = \boxed{-18}$

b) $\vec{A} = 3\hat{i} - 2\hat{j}$ $\quad \vec{B} = 6\hat{i} + 4\hat{j}$

$\vec{A} \cdot \vec{B} = (3)(6) + (-2)(4) = \boxed{10}$

13. a) $\vec{A} = \sqrt{(3)^2 + (4)^2} = 5$ $\quad \vec{B} = \sqrt{(2)^2 + (-6)^2} = 6.32$

$\vec{A} \cdot \vec{B} = AB\cos\theta \Rightarrow -18 = (5)(6.32)\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{-18}{(5)(6.32)}\right) = \boxed{125^\circ}$

b) $\vec{A} = \sqrt{(3)^2 + (2)^2} = 3.61$ $\quad \vec{B} = \sqrt{(6)^2 + (4)^2} = 7.21$

$\vec{A} \cdot \vec{B} = AB\cos\theta \Rightarrow 10 = (3.61)(7.21)\cos\theta \Rightarrow \theta = \cos^{-1}\left(\frac{10}{(3.61)(7.21)}\right) = \boxed{67^\circ}$

15. a) $\vec{A} \cdot \vec{B} = AB\cos\theta = (4)(2)\cos 110^\circ = \boxed{-2.74}$

b) $\vec{C} \cdot \vec{D} = CD\cos\theta = (4)(5)\cos 180^\circ = \boxed{-20}$

c) $\vec{E} \cdot \vec{F} = EF\cos\theta = (4)(3)\cos 30^\circ = \boxed{10.39}$

17. a) $\vec{W} = \vec{F} \cdot \Delta\vec{r} = (0.04\hat{i} - 0.06\hat{j}) \cdot (2.0\hat{i} - 2.0\hat{j}) = (0.08 + 0.12) = \boxed{0.20 J}$

b) $W = \Delta K = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}mv_f^2 = v_f = \sqrt{\frac{2W}{m}}$

$\Rightarrow v_f = \sqrt{\frac{2(0.20J)}{0.045kg}} = \boxed{2.98\ m/s}$

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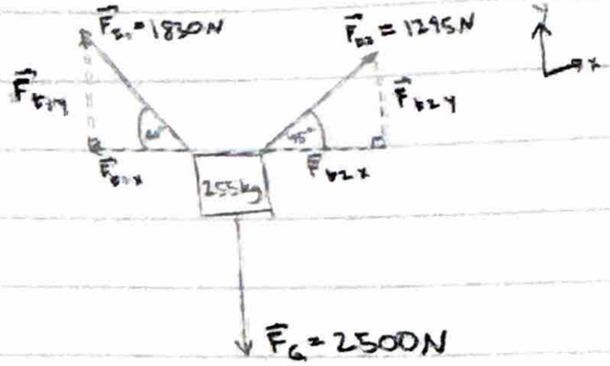
18.



$\frac{6}{6}$

$W_3 = \vec{F}_G \cdot \Delta r = 2500N \cdot 5m = \boxed{12500J}$

$W_2 = \vec{F}_{T_2} \cdot \Delta r = 1295N \cdot 5m \cdot \cos 135° = \boxed{-4578.52J}$

$W_1 = \vec{F}_{T_1} \cdot \Delta r = 1830N \cdot 5m \cdot \cos 150° = \boxed{-7924.13J}$

20. $W = \int_{s_i}^{s_f} F_s \, ds$ = area under the force curve

a) $W = (4N)(1m - 0m) = \boxed{4J}$

b) $W = (4N)(0.5m) + (-4N)(0.5m) = \boxed{0J}$

c) $W = \frac{1}{2}(-4N)(1m) = \boxed{-2J}$

21. $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W = \int_{x_i}^{x_f} F_x \, dx$ = area under the curve

$\frac{4}{4}$ $\frac{1}{2}mv_f^2 - \frac{1}{2}(0.5kg)(2m/s)^2 = W$

$v_f = \sqrt{\frac{2(W+1)}{m}} = \sqrt{\frac{2(15 + \frac{1}{2} \cdot 15 - 2 + 1)}{0.5}} = \boxed{11.14 m/s}$

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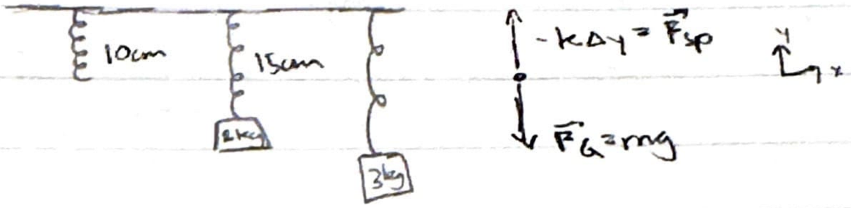
27.



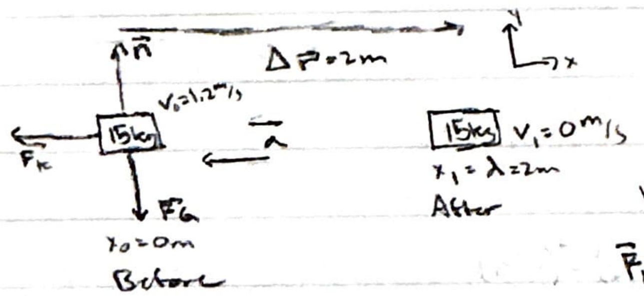
a) $F_{sp} = -k\Delta y = mg$ (because at rest)

$\Rightarrow k = -\left(\frac{mg}{\Delta y}\right) = -\left(\frac{2kg \cdot 9.8 m/s^2}{-0.15m - (-0.1m)}\right) = \boxed{392 N/m}$

b) $F_{sp} = -k\Delta y = mg$

$\Rightarrow y_f = \frac{mg}{-k} + y_i = \frac{3kg \cdot 9.8 m/s^2}{-392N} - 0.1m = -0.175m \Rightarrow \boxed{17.5cm}$

33.



$\vec{F}_G = \vec{n} = 147N$

$\vec{F}_k = \mu_k n = m\vec{a}$

$W_{net} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \Rightarrow$

$\vec{F}_{net} \cdot \Delta\vec{r} = \vec{f}_k \cdot \Delta\vec{r} = 0J - \frac{1}{2}mv_i^2 = -30J$

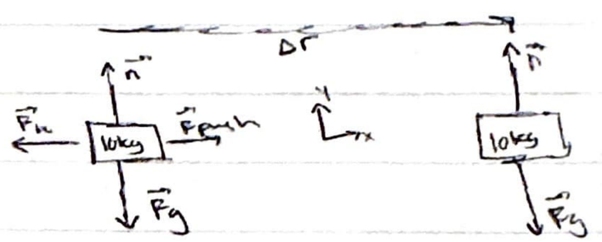
$(f_k)(d)\cos 180° = -\frac{1}{2}mv_i^2 - \mu_k mgd = -\frac{1}{2}mv_0^2 \Rightarrow \mu_k = \frac{v_0^2}{2gd}$

$\frac{1}{2}mv_0^2 = \mu_k mgd \Rightarrow \mu_k = \frac{v_0^2}{2gd}$ Work Cards Energy

$\mu_k = \frac{(1.2 m/s)^2}{2(9.8)(2)} = \boxed{0.037}$ you can also use kinematics

10/10

37.



$\Delta r = 1.0 m/s (3s) = 3m$

$\mu_k = 0.6$ $t = 3s$

$v_0 = v_1 = 1 m/s$ $a = 0 m/s^2$

a) $\vec{F}_{net} = \vec{F}_{push} - \vec{F}_k = 0N \Rightarrow \vec{F}_{push} = \mu_k n = (0.6)(10kg)(9.8 m/s^2) = 58.8N$

$W = F \cdot d \Rightarrow W = 58.8N(3m) = \boxed{176.4J}$

b) $P = \frac{W}{t} = \frac{176.4J}{3s} = \boxed{58.8 W}$

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40. a) $x_f = x_0 + v_0 \Delta t + \frac{1}{2} a (\Delta t)^2$

$50m = 0m + 0m/s(7s) + \frac{1}{2} a (7s)^2 = a = 2.04\, m/s^2$

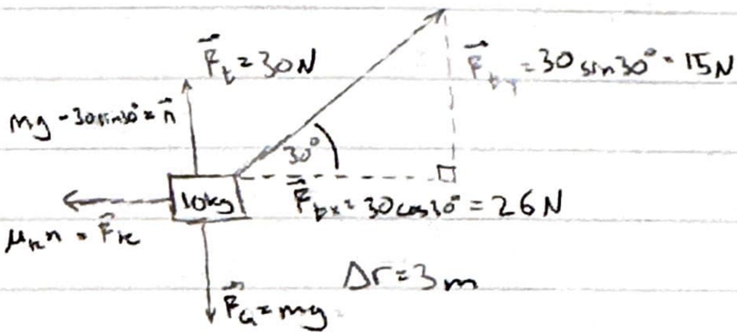
$F = ma = 50kg \times 2.04\, m/s^2 = \boxed{102N}$

b) $P = \vec{F} \cdot \vec{v} \Rightarrow P(2s) = 102N \cdot (2 \cdot 2.04) = \boxed{416.16W}$

$P(4s) = 102N \cdot (4 \cdot 2.04) = \boxed{832.32W}$

$P(6s) = 102N \cdot (6 \cdot 2.04) = \boxed{1248.48W}$

45.



$\vec{F}_k = \mu_k n = \mu_k (mg - \vec{F}_{ty}) = 0.2(10 \cdot 9.8 - 15) = 16.6N$

$\vec{F}_{net} = \vec{F}_{tx} - \vec{F}_k = 26N - 16.6N = 9.4N$

$W = F \cdot d = 9.4N(3m) = 28.2J = \Delta K = \frac{1}{2} m v_f^2$

$v_f = \sqrt{\frac{2(28.2)}{10}} = \boxed{2.37\, m/s}$

56. $-k = \frac{F}{\Delta s} \Rightarrow -k = \frac{m_1 g}{\Delta y_1} = \frac{(m_1 + m_2) g}{\Delta y_1 + \Delta y_2} \Rightarrow m_2 = m_1 \left(\frac{\Delta y_1 + \Delta y_2}{\Delta y_1} - 1 \right)$

$\Rightarrow (65kg) \left(\frac{5.5cm + 4.5cm}{5.5cm} - 1 \right) = \boxed{53kg}$

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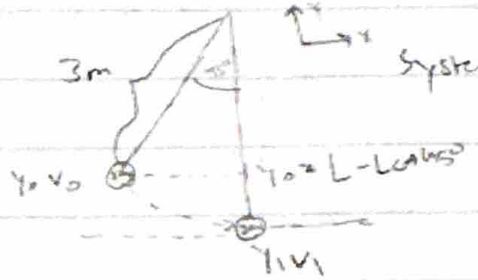
0: Special Instructions For Energy Problems!
Write down the system you define

20/20

Ch. 10 E.P. # 10, 11, 22, 23, 24, 28, 41, 44, 49, 52, 54, 64

10.



System: child + Earth

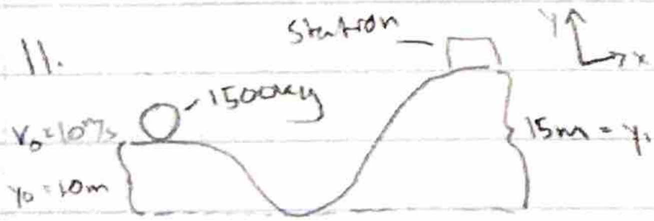
$E_i = E_f$

$U_i + \cancel{K_i}^0 = \cancel{U_f}^0 + K_f$

$mgy_0 = \frac{1}{2}mv_f^2$

$\boxed{4.15\ m/s} = \sqrt{(2)(9.8\ m/s^2)(3 - 3\cos 45°\ m)} \Leftarrow v_f = \sqrt{2gy_0}$

11.



System: car + Earth

$E_i = E_f$

$U_i + K_i = U_f + K_f$

$mgy_0 + \frac{1}{2}mv_0^2 = mgy_f + \frac{1}{2}mv_f^2$

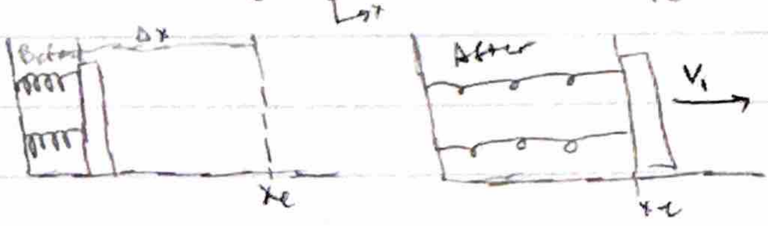
$v_f = \sqrt{2(gy_0 + \frac{1}{2}v_0^2 - gy_f)} = \sqrt{2(9.8 \times 10 + \frac{1}{2}(10\ m/s)^2 - 9.8 \times 15)} = \boxed{1.4\ m/s}$

22. a)

$x_f = x_e = 0$

$x_0 - x_e = \Delta x$

b)



System: Spring + Block

$E_i = E_f$

$U_i + \cancel{K_i}^0 = \cancel{U_f}^0 + K_f$

$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_f^2$

$v_f = v_0 \Rightarrow \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_0^2 \rightarrow$ for one spring

$2(\frac{1}{2}k(\Delta x)^2) = \frac{1}{2}mv_f^2$

$v_f = v_1 \Rightarrow 2(\frac{1}{2}k(\Delta x)^2) = \frac{1}{2}mv_1^2 \rightarrow$ for two springs

$2(\frac{1}{2}mv_0^2) = \frac{1}{2}mv_1^2$

$\cancel{m}v_0^2 = \frac{1}{2}\cancel{m}v_1^2$

$2v_0^2 = v_1^2$

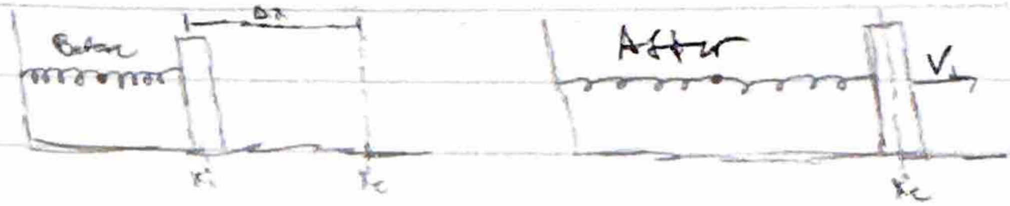
$\boxed{v_1 = \sqrt{2}\, v_0}$

$\frac{10}{10}$

23. a)

System: Spring + Block

b)



$E_i = E_f \Rightarrow U_i + \cancel{K_i}^{0} = \cancel{U_f}^{0} + K_f \Rightarrow U_i = K_f \Rightarrow \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_f^2$

$\Rightarrow$ one spring $= \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv_0^2$

$\Rightarrow$ two springs $= \frac{1}{2}k\left(\frac{\Delta x}{2}\right)^2 + \frac{1}{2}k\left(\frac{\Delta x}{2}\right)^2 = \frac{1}{2}mv_f^2$

$\Rightarrow 2\left(\frac{1}{2}k\frac{(\Delta x)^2}{4}\right) = \frac{1}{2}mv_1^2 \Rightarrow \frac{1}{2}\left(\frac{1}{2}k(\Delta x)^2\right) = \frac{1}{2}mv_1^2$

$\Rightarrow \frac{1}{2}\left(\frac{1}{2}mv_0^2\right) = \frac{1}{2}mv_1^2 \Rightarrow \boxed{v_1 = \frac{1}{\sqrt{2}}V_0}$

24. a) $F_x = -\frac{dU}{dx} \Rightarrow$ At $x = 1.0\,m$, $F > 0$, so it speeds up to the $\boxed{\text{right}}$.

b) Max speed = local min $\Rightarrow 0 = -\frac{dU}{dx} \Rightarrow$ at $\boxed{x = 4}$

$K = E - U \Rightarrow 4J - 1J = 3J$

$K = \frac{1}{2}mv^2 \Rightarrow 2K = mv^2$

$\Rightarrow \frac{2K}{m} = v^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3)}{0.02\,kg}} = \boxed{17.3\,m/s}$

c) Turning point is when $K = 0$ and $U = E$

$\Rightarrow$ This happens at $\boxed{x = 1.0\,m}$ and $\boxed{x = 6.0\,m}$

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28. a) The turning points occur when $K=0$ and $E=U$.

$E=12J \Rightarrow \boxed{x=1m}$ and $\boxed{x=8m}$

b) $E=U+K \Rightarrow K=E-U \Rightarrow 12J-8J=4J$

$K=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(4J)}{0.5kg}}=\boxed{4\,m/s}$

c) minimum V occurs at $\boxed{x=6m}$

$K=E-U=12J \Rightarrow \frac{1}{2}mv^2=12J \Rightarrow v=\sqrt{\frac{(2)(12)}{0.5kg}}=\boxed{6.9\,m/s}$

d) If the particle has 4J then it can reach $x=2m$ but not $x=4$ because it requires 8J of Energy.

41. $E_i=E_f$

$\cancel{U_i}^0+K_i=U_f+K_f$

$\frac{1}{2}mv_0^2=mgy_f+\frac{1}{2}mv_f^2$

$y_1 v_1 = 3\,m/s$ System: Bee + Earth

R = 10cm $y_{top}=y_f=2R=0.2m$

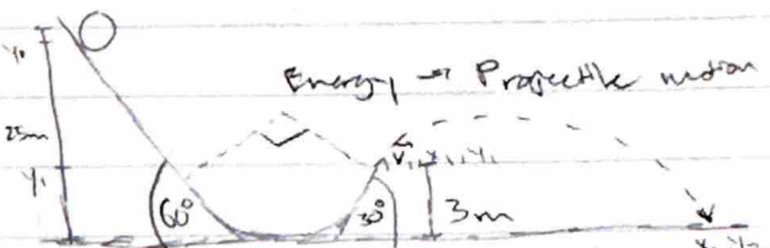
$y=R-R\cos\theta$

$\frac{1}{2}v_0^2-gy_f=\frac{1}{2}v_f^2 \Rightarrow v_f=\sqrt{2(\frac{1}{2}v_0^2-gy_f)}=\sqrt{2(\frac{1}{2}v_0^2-g(2r))}$

$=v_f=\sqrt{2(\frac{1}{2}\cdot(3m/s)^2-9.8m/s^2(0.2m))}=\boxed{2.25\,m/s}$

44.

Energy → Projectile motion



$E_i=E_f$ System: Skier + Earth

$U_i+\cancel{K_i}^0=U_f+K_f$

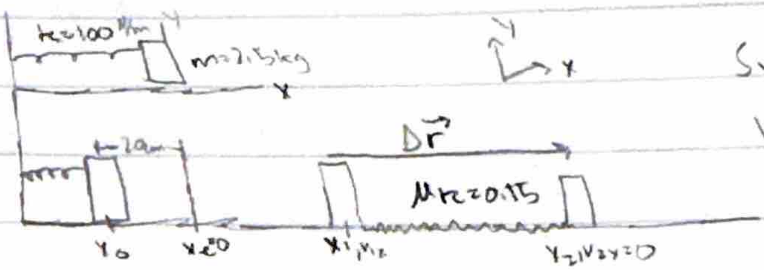
$mgy_0=mgy_f+\frac{1}{2}mv_f^2$

$v=\sqrt{2g(y_0-y_f)}=\sqrt{2(9.8)(25-3)}=20.77\,m/s$

$\Rightarrow y_f=y_0+v_{0y}\Delta t-\frac{1}{2}g(\Delta t)^2 \Rightarrow 0=3+v\sin 30°\Delta t-\frac{1}{2}g(\Delta t)^2=\Delta t=2.37$

$\Rightarrow x_f=x_0+v_{0x}\Delta t \Rightarrow x_f=(v_0\cos 30°)\Delta t=\boxed{42.7m}$

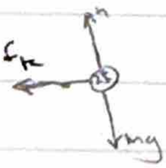
49.



System: Box + Spring

$W_{friction} = \Delta E = \Delta U + \Delta K$

$U_i + K_i + W_{friction} = U_f + K_f$



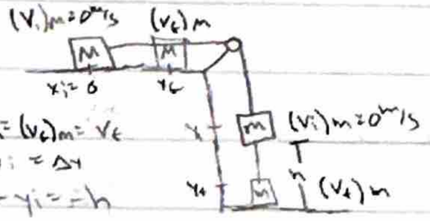
$W_{friction} = \vec{f_k} \cdot \Delta\vec{s} = (\mu_k n)\,\Delta x \cos 180 = -(\mu_k mg)(x_f - x_i) = -\mu_k mg x_f$

$E_i + W_{friction} = E_f \Rightarrow U_i + \cancel{K_i}^0 + W_{friction} = \cancel{U_f}^0 + \cancel{K_f}^0$

$\frac{1}{2}k(x_i - x_0)^2 - (\mu_k mg)x_f = 0 \Rightarrow x_f = \frac{1}{2}\frac{k(-x_0)^2}{\mu_k mg}$

$x_f = \frac{1}{2}\frac{(100)(-0.2)^2}{(0.15)(2.5)(9.8)} = \boxed{0.544\,m}$

52.



System: Blocks + Earth

$(v_f)_M = (v_f)_m = v_f$

$x_f - x_i = \Delta x$

$y_f - y_i = -h$

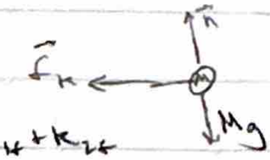
a) $E_i + W_{friction} = E_f \Rightarrow$

$U_{2i} + \cancel{K_{1i}}^0 + \cancel{K_{2i}}^0 + W_{friction} = \cancel{U_{2f}}^0 + K_{1f} + K_{2f}$

$U_{2i} + W_{friction} = K_{1f} + K_{2f}$

$mgh - \mu_k Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}mv_f^2$

$\boxed{v_f = \sqrt{\frac{(2gh)(m - \mu_k M)}{M+m}}}$



$W_{friction} = \vec{f_k} \cdot \Delta\vec{s}$

$W_{friction} = (\mu_k n)\,h \cos 180°$

$W_{friction} = -\mu_k Mgh$

b) no friction $\Rightarrow \mu_k = 0$

$\boxed{v_f = \sqrt{\frac{2ghm}{M+m}}}$

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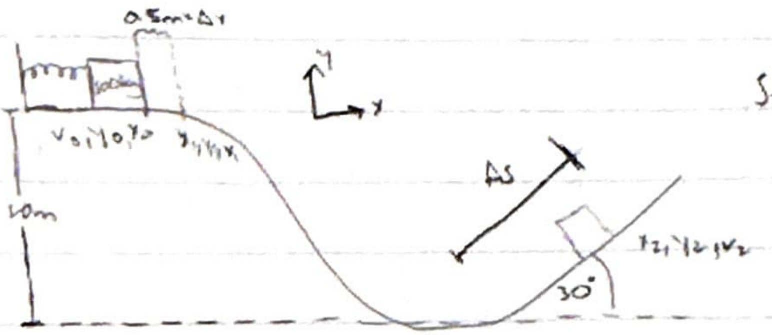
54.



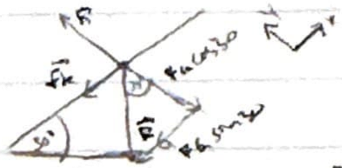
System: Earth + Student + Spring

10/10

a) $E_i = E_f \Rightarrow U_i + \cancel{K_i}^{0} = \cancel{U_f}^{0} + K_f \Rightarrow U_i = K_f \Rightarrow \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{k(\Delta x)^2}{m}} = \sqrt{\frac{80{,}000(0.5m)^2}{100kg}} = \boxed{14.14 \text{ m/s}}$

b)



$W_{friction} = \vec{f_k} \cdot \vec{\Delta s}$

$W_{friction} = (\mu_k n)\Delta s \cos 180°$

$U_i + K_i + W_{friction} = U_f + \cancel{K_f}^{0}$ $\quad W_{friction} = -(\mu_k mg\cos 30°)\Delta s$

$mgy_1 + \frac{1}{2}mv_i^2 - (\mu_k mg\cos 30°)\Delta s = mgy_2 \quad y_2 = \Delta s \sin 30°$

$\Delta s = \frac{2gy_1 + v_i^2}{2g(\mu_k \cos 30° + \sin 30°)} = \frac{2(9.8)(10) + 14.14^2}{2(9.8)(0.15\cos 30° + \sin 30°)} = \boxed{32.1 \text{ m}}$

64. a) $\vec{F} = (2xy\hat{i} + 3y\hat{j})$ N

$(0,0) \to (a,0) \to (a,b) \Rightarrow W = \left[\int_0^a F_x dx\right]_{y=0} + \left[\int_0^b F_y dy\right]_{x=a}$

$W = \cancel{\left[\int_0^a (0)dx\right]_{y=0}} + \left[\int_0^b (3y)dy\right]_{x=a} = \left[\frac{3}{2}y^2\right]_0^b = \boxed{\frac{3}{2}b^2}$

b) $(0,0) \to (0,b) \to (a,b) \quad W = \left[\int_0^b F_y dy\right]_{x=0} + \left[\int_0^a F_x dx\right]_{y=b}$

$W = \left[\int_0^b (3y)dy\right]_{x=0} + \left[\int_0^a (2bx)dx\right]_{y=b} = \left[\frac{3}{2}y^2\right]_0^b + \left[bx^2\right]_0^a$

$= \boxed{\frac{3}{2}b^2 + a^2 b}$

c) No, different answers for different paths so not conservative.